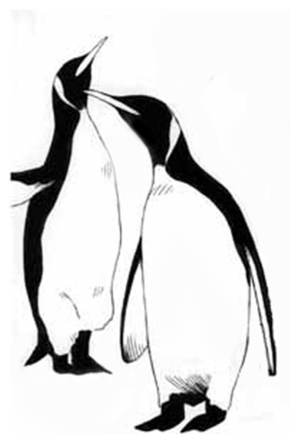
Commonsense Reasoning through Circumscription:

What about the Penguins?



by Mark Rich '98

What is Commonsense?

- **Rules of Thumb**

  Glue is sticky, birds can fly, three-year-olds cannot drive.

- **Flexible**

  New information can change these rules. What if the glue is 200 years old?

- **Uncomplicated**

  We use commonsense everyday, and it is instantaneous.

- **Minimal**

  We want the least number of exceptions to our rules.

Birds Can Fly

☹ Universally Quantify

$$\forall x\ ((Bird(x) \rightarrow Flies(x))$$

☹ But Penguins can't fly . . .

$$\forall x\ ((Bird(x) \wedge \neg\ Penguin(x) \wedge$$
$$\neg\ Ostrich(x) \wedge \neg\ Dead(x)) \rightarrow Flies(x))$$

☺ Abnormality Predicate

$$\forall x\ ((Bird(x) \wedge \neg\ Ab(x)) \rightarrow Flies(x))$$

Approaches to Modeling

_ First-Order Logic

possible in principle, not in practice
not flexible
no minimization

_ Probabilistic Logic

too complex
no minimization

_ Nonmonotonic Logic

rules but flexible
uncomplicated
becomes minimal with
circumscription

What is Nonmonotonic Logic?

- **Monotonic Logic**

  If $\Gamma \models S$ then $\Gamma \cup \Delta \models S$

- **Nonmonotonic Logic**

  $\Gamma \models S$
  but for some Δ,
  it is not the case that
  $\Gamma \cup \Delta \models S$

- **Informally . . .**

  There is at least one new axiom which will invalidate our conclusion S.

The Blocks World

✵ Axioms

A= $\{\forall x\ ((Block(x) \wedge \neg Ab(x)) \rightarrow Ontable(x))$

$$Block(B_1) \wedge Block(B_2)$$

$$\neg Ontable(B_1)$$

$$B_1 \neq B_2\ \}$$

✵ We want to prove

$$Ontable(B_2)$$

✵ But we need more . . .

It is still consistent if $\neg Ontable(B_2)$

We need $\neg Ab(B_2)$

The Extent of Predicates

Objects: *a*, *b*, *c*, *d*

$Rectangle(a) \wedge Rectangle(b) \wedge$

$Rectangle(c) \wedge Rectangle(d)$

$Square(a) \wedge Square(c)$

$Square(x)$ is a proper subset of $Rectangle(x)$

i.e. $Square(x) < Rectangle(x)$

❖ Add Circumscription

Rules of Thumb: Axiom set A

Flexible: Circumscription is reanalyzed each time we gain new information

Uncomplicated: It is only one axiom

Minimal: It finds the minimum exceptions to our rules of thumb.

Circumscription

❄ Explanation

We will circumscribe the axiom set **A**, and minimize the predicate *P*. To minimize *P*, means to minimize the extent for which *P* is true. There are no proper subsets of *P*.

❄ Formal Definition

$$\mathbf{CIRC}[\boldsymbol{A}(\boldsymbol{P});\ \boldsymbol{P}]:$$

$$A(P) \wedge \neg\exists p\ [A(p) \wedge p < P]$$

Examples of Circumscription

Example 1: $A = \{P(a)\}$

CIRC[*A*; *P*]:

$$\forall x[P(x) \equiv x = a]$$

Example 2: $A = \{P(a)\ , P(b)\}$

CIRC[*A*; *P*]:

$$\forall x[P(x) \equiv (x = a \vee x = b)]$$

Example 3: $A = \{Block(a)\}$

CIRC[*A*; *Block*]:

$$\forall x[Block(x) \equiv x = a]$$

Back to Blocks

A= $\{\forall x\, ((Block(x) \wedge \neg Ab(x)) \rightarrow Ontable(x))$

$Block(B_1) \wedge Block(B_2)$

$\neg Ontable(B_1)$

$B_1 \neq B_2\ \}$

CIRC[*A*; *Ab*; *Ontable*]: $\forall x\, (Ab(x) \equiv x = B_1)$

_ Proof of $Ontable(B_2)$:

Using $B_2 \neq B_1$ and $\forall x\, (Ab(x) \equiv x = B_1)$,
we know that $\neg Ab(B_2)$.
Now we know $Block(B_2) \wedge \neg Ab(B_2)$
Therefore, $Ontable(B_2)$

Opus Can Fly

A= $\{ \; \forall x\, ((Bird(x) \wedge \neg Ab(x)) \rightarrow Flies(x))$

$$Bird(Tweety) \wedge Bird(Opus)$$

$$\neg Flies(Tweety)$$

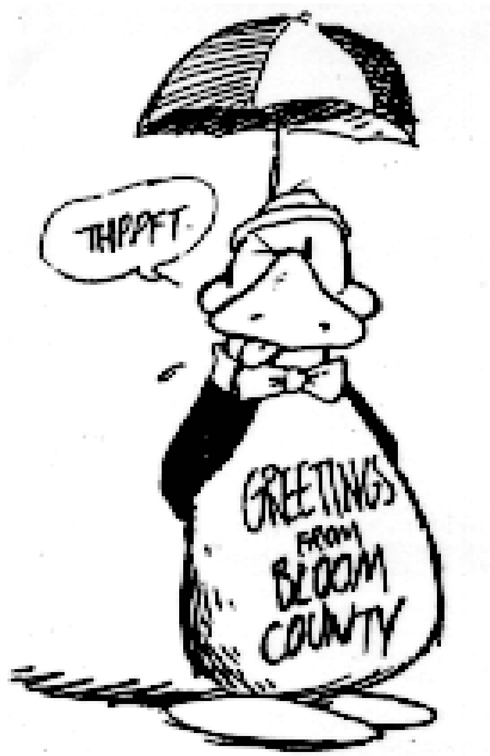
$$Tweety \neq Opus \; \}$$

CIRC[*A*; *Ab*; *Flies*]:

$$\forall x\, (Ab(x) \equiv (x = Tweety))$$

- **As in the blocks world, we can now prove** *Flies*(*Opus*)

But Opus is a Penguin!

Opus = 

Fred Can Fly

A= { $\forall x\,((Bird(x) \wedge \neg Ab(x)) \rightarrow Flies(x))$

$$Bird(Tweety) \wedge Bird(Opus)$$

$$\neg Flies(Tweety)$$

$$Tweety \neq Opus$$

$$\neg Flies(Opus)$$

$$Bird(Fred)$$

$$Opus \neq Fred$$

$$Tweety \neq Fred$$ }

CIRC[*A*; *Ab*; *Flies*]:

$$Ab(x) \equiv (x = Tweety \vee x = Opus)$$

_ **We can prove** $Flies(Fred)$

Commonsense Reasoning through Circumscription:

What about Dead Penguins?

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Properties of Commonsense

Rules of Thumb
Flexible
Uncomplicated
Minimal

Logic Notation

$\forall$ = for all (universal quantifier)
$\exists$ = there exists (existential quanifier)
$\wedge$ = and (conjunction)
$\vee$ = or (disjunction)
$\neg$ = not (negation)
$\rightarrow$ = implies
$\equiv$ = is equivalent to
_ = we can prove
$\cup$ = unioned with

Monotonic Logic

If you have a set of axioms Γ by which you can prove a statement S, then no matter what other axioms Δ you add to your set, you will always be able to prove S.

Nonmonotonic Logic

Contrary to monotonic logic, there exists some axioms Δ such that adding them to our set of axioms Γ invalidates our proof of S. For example: learning new information about Tweety being an ostrich invalidates our conclusion that Tweety can Fly.

If we're not told that it's true, then we can conclude that it's false.

Circumscription

Rules of Thumb: Axiom set A

Flexible: The circumscription axiom is reanalyzed each time new axioms are added to A.

Uncomplicated: There is only one axiom added for each predicate to circumscribe.

Minimal: The circumscription axiom finds the minimum exceptions to our default rule.

Basic Axiom of Circumscription

$CIRC[A(P); P]: A(P) \wedge \neg\exists p\ [A(p) \wedge p < P]$

We circumscribe the axiom set A containing P, and minimize P.

Circumscription says that there does not exist *p* such that if we substitute *p* into *A* for *P*, then *p* would have a lesser extent than *P*.

$p < P$ means that the extent of *p* is a proper subset of the extent of *P*.

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