1 Wisdom of Crowds

1.1 Numerical Wisdom

\[ \text{average} = \frac{\text{sum of answers}}{\text{total}} \]

\[ \text{collective error} = (\text{average guess} - \text{correct answer})^2 \]

\[ \text{individual error} = (\text{guess} - \text{correct answer})^2 \]

\[ \text{individual diversity} = (\text{guess} - \text{average guess})^2 \]

\[ \text{collective error} = \text{average individual error} - \text{average diversity} \]

1.2 Majority Wisdom

This is Pascal’s triangle, where each number is the sum of the two numbers above it.

\[
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1 \\
1 & 4 & 6 & 4 & 1 \\
& & & & & & . . . . . . . . . .
\end{array}
\]
If we do not care about the ordering of the choices, only the elements, we can define \textbf{combinations} as \(\binom{n}{k}\), which is read “n choose k”.

\[
\binom{n}{k} = \frac{n!}{(n-k)!k!}
\]

\(\binom{n}{k}\) is also equal to the \(k + 1\)th number on the \(n + 1\)th row of Pascal’s Triangle.

Using combinations, we now have a tool to calculate the probability that an event with probability \(p\) will occur exactly \(k\) times in an experiment repeated \(n\) times.

\[
\binom{n}{k} p^k (1 - p)^{n-k}
\]

And therefore assuming \(n\) is odd, the probability that the majority is correct when each individual is correct with probability \(p\) is

\[
\sum_{k=\lceil \frac{n}{2} \rceil}^{n} \binom{n}{k} p^k (1 - p)^{n-k}
\]