2 Binary Numbers

We are most familiar with decimal numbers written in base 10 notation, such that there is a ones place, tens place, hundreds place, etc. The language of computers, and some other branches of mathematics, is in binary numbers. Instead of each digit being one of the numbers {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, our choices are now {0, 1}. This means our digit places are 1, 2, 4, 8, 16, etc. We will denote binary numbers with a subscript of 2, to differentiate 100 (one hundred) from 100_2 (four).

2.1 Binary to Decimal

Every decimal number can be written as an expansion of digits times powers of 10. For example

$$1729 = 1 * 1000 + 7 * 100 + 2 * 10 + 9 * 1$$

or

$$1729 = 1 * 10^3 + 7 * 10^2 + 2 * 10^1 + 9 * 10^0$$

Binary numbers can be expanded the same way, except instead of powers of 10, we have powers of 2.

$$10110_2 = 1 * 2^4 + 0 * 2^3 + 1 * 2^2 + 1 * 2^1 + 0 * 2^0$$

or

$$\begin{aligned} 10110_2 &= 1 * 16 + 0 * 8 + 1 * 4 + 1 * 2 + 0 * 1 \\ &= 16 + 4 + 2 \\ &= 22 \end{aligned}$$

2.2 Decimal to Binary

To convert a decimal number n to a binary number

- Find the largest power of two (p) which is less than or equal to the number
- Now repeat the following steps while the number $p >= 1$:

A: If $p <= n$, write a 1, otherwise a 0, then subtract p from n

B: Divide p by 2

For example, we will convert the number 47. 32 is the largest power of 2 smaller than 47, so $p = 32$.

32 $<$ 47, so write a 1

n now equals 47 - 32 = 15

p now equals 32 / 2 = 16

16 is not $<$ 15, so write a 0

p now equals 16 / 2 = 8

8 $<$ 15, so write a 1

n now equals 15 - 8 = 7

p now equals 8 / 2 = 4

4 $<$ 7, so write a 1

n now equals 7 - 4 = 3

p now equals 4 / 2 = 2

2 $<$ 3, so write a 1

n now equals 3 - 2 = 1

p now equals 2 / 2 = 1

1 $<$ 2, so write a 1

n now equals 1 - 1 = 0

p now equals 1 / 2 = 0.5, so we stop.

Our resulting binary number for 47 is 101111_2