CSC 107: Course Notes

Mark Goadrich

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1 Wisdom of Crowds

1.1 Numerical Wisdom

$$average = \frac{sum\ of\ answers}{total}$$

$$collective\ error = (average\ guess - correct\ answer)^2$$

$$individual\ error = (guess - correct\ answer)^2$$

$$individual\ diversity = (guess - average\ guess)^2$$

$$collective\ error = average\ individual\ error - average\ diversity$$

1.2 Majority Wisdom

This is **Pascal's triangle**, where each number is the sum of the two numbers above it.

$$\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1 \\
. & . & . & . & . & . & . & . & .
\end{array}$$

If we do not care about the ordering of the choices, only the elements, we can define **combinations** as $\binom{n}{k}$, which is read "n choose k".

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

$\binom{n}{k}$ is also equal to the $k+1$th number on the $n+1$th row of Pascal's Triangle.

Using combinations, we now have a tool to calculate the probability that an event with probability p will occur exactly k times in an experiment repeated n times.

$$\binom{n}{k} p^k (1-p)^{n-k}$$

And therefore assuming n is odd, the probability that the majority is correct when each individual is correct with probability p is

$$\sum_{k=\lceil \frac{n}{2} \rceil}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$