Chapter 6

Fruitful functions

6.1 Adding new functions

So far, we have only been using the functions that come with Python, but it is also possible to add new functions. A function definition specifies the name of a new function and the sequence of statements that execute when the function is called.

Here is an example:

```python
def print_lyrics():
    print "I'm a lumberjack, and I'm okay."
    print "I sleep all night and I work all day."
```

def is a keyword that indicates that this is a function definition. The name of the function is print_lyrics. The rules for function names are the same as for variable names: letters, numbers and some punctuation marks are legal, but the first character can’t be a number. You can’t use a keyword as the name of a function, and you should avoid having a variable and a function with the same name.

The empty parentheses after the name indicate that this function doesn’t take any arguments.

The first line of the function definition is called the header; the rest is called the body. The header has to end with a colon and the body has to be indented. By convention, the indentation is always four spaces. The body can contain any number of statements.

The strings in the print statements are enclosed in double quotes. Single quotes and double quotes do the same thing. Most people use single quotes except in cases like this where a single quote (which is also an apostrophe) appears in the string.

If you type a function definition in interactive mode, the interpreter prints ellipses (…) to let you know that the definition isn’t complete:
>>> def print_lyrics():
...    print "I'm a lumberjack, and I'm okay."
...    print "I sleep all night and I work all day."
...

To end the function, you have to enter an empty line (this is not necessary in a script).

Defining a function creates a variable with the same name.

>>> print print_lyrics
<function print_lyrics at 0xb7e99e9c>
>>> print_type(print_lyrics)
<type 'function'>

The value of print_lyrics is a **function object**, which has type function.

The syntax for calling the new function is the same as for built-in functions:

>>> print_lyrics()
I'm a lumberjack, and I'm okay.
I sleep all night and I work all day.

Once you have defined a function, you can use it inside another function. For example, to repeat the previous refrain, we could write a function called repeat_lyrics:

def repeat_lyrics():
    print_lyrics()
    print_lyrics()

And then call repeat_lyrics:

>>> repeat_lyrics()
I'm a lumberjack, and I'm okay.
I sleep all night and I work all day.
I'm a lumberjack, and I'm okay.
I sleep all night and I work all day.

But that's not really how the song goes.

### 6.2 Definitions and uses

Pulling together the code fragments from the previous section, the whole program looks like this:

def print_lyrics():
    print "I'm a lumberjack, and I'm okay."
    print "I sleep all night and I work all day."

...
6.3. Flow of execution

```python
def repeat_lyrics():
    print_lyrics()
    print_lyrics()

repeat_lyrics()
```

This program contains two function definitions: `print_lyrics` and `repeat_lyrics`. Function definitions get executed just like other statements, but the effect is to create the new function. The statements inside the function do not get executed until the function is called, and the function definition generates no output.

As you might expect, you have to create a function before you can execute it. In other words, the function definition has to be executed before the first time it is called.

**Exercise 6.1.** Move the last line of this program to the top, so the function call appears before the definitions. Run the program and see what error message you get.

**Exercise 6.2.** Move the function call back to the bottom and move the definition of `print_lyrics` after the definition of `repeat_lyrics`. What happens when you run this program?

6.3 Flow of execution

In order to ensure that a function is defined before its first use, you have to know the order in which statements are executed, which is called the **flow of execution**.

Execution always begins at the first statement of the program. Statements are executed one at a time, in order from top to bottom.

Function definitions do not alter the flow of execution of the program, but remember that statements inside the function are not executed until the function is called.

A function call is like a detour in the flow of execution. Instead of going to the next statement, the flow jumps to the body of the function, executes all the statements there, and then comes back to pick up where it left off.

That sounds simple enough, until you remember that one function can call another. While in the middle of one function, the program might have to execute the statements in another function. But while executing that new function, the program might have to execute yet another function!

Fortunately, Python is adept at keeping track of where it is, so each time a function completes, the program picks up where it left off in the function that called it. When it gets to the end of the program, it terminates.

What’s the moral of this sordid tale? When you read a program, you don’t always want to read from top to bottom. Sometimes it makes more sense if you follow the flow of execution.
6.4 Parameters and arguments

Some of the built-in functions you have used require arguments. For example, when you call `math.sin` you pass a number (in radians) as an argument. Some functions take more than one argument; `math.pow` takes two, the base and the exponent.

Inside the function, the arguments are assigned to variables called parameters. Here is an example of a user-defined function that takes an argument:

```python
def print_twice(bruce):
    print bruce
    print bruce
```

This function assigns the argument to a parameter named `bruce`. When the function is called, it prints the value of the parameter, whatever it is, twice.

This function works with any value that can be printed.

```python
>>> print_twice('Spam')
Spam
Spam
>>> print_twice(17)
17
17
>>> print_twice(math.pi)
3.141592653589793
3.141592653589793
```

The same rules of composition that apply to built-in functions also apply to user-defined functions, so we can use any kind of expression as an argument for `print_twice`:

```python
>>> print_twice('Spam '*4)
Spam Spam Spam Spam
Spam Spam Spam Spam
>>> print_twice(math.cos(math.pi))
-1.0
-1.0
```

The argument is evaluated before the function is called, so in the examples the expressions 'Spam '*4 and `math.cos(math.pi)` are only evaluated once.

You can also use a variable as an argument:

```python
>>> michael = 'Eric, the half a bee.'
>>> print_twice(michael)
Eric, the half a bee.
Eric, the half a bee.
```

The name of the variable we pass as an argument (`michael`) has nothing to do with the name of the parameter (`bruce`). It doesn’t matter what the value was called back home (in the caller); here in `print_twice`, we call everybody `bruce`. 
When you create a variable inside a function, it is **local**, which means that it only exists inside the function. For example:

```python
def cat_twice(part1, part2):
    cat = part1 + part2
    print_twice(cat)
```

This function takes two arguments, concatenates them, and prints the result twice. Here is an example that uses it:

```python
>>> line1 = 'Bing tiddle '
>>> line2 = 'tiddle bang.'
>>> cat_twice(line1, line2)
Bing tiddle tiddle bang.
Bing tiddle tiddle bang.
```

When `cat_twice` terminates, the variable `cat` is destroyed. If we try to print it, we get an exception:

```python
>>> print cat
NameError: name 'cat' is not defined
```

Parameters are also local. For example, outside `print_twice`, there is no such thing as `bruce`.

### 6.6 Stack diagrams

To keep track of which variables can be used where, it is sometimes useful to draw a **stack diagram**. Like state diagrams, stack diagrams show the value of each variable, but they also show the function each variable belongs to.

Each function is represented by a **frame**. A frame is a box with the name of a function beside it and the parameters and variables of the function inside it. The stack diagram for the previous example looks like this:
The frames are arranged in a stack that indicates which function called which, and so on. In this example, print_twice was called by cat_twice, and cat_twice was called by __main__, which is a special name for the topmost frame. When you create a variable outside of any function, it belongs to __main__.

Each parameter refers to the same value as its corresponding argument. So, part1 has the same value as line1, part2 has the same value as line2, and bruce has the same value as cat.

If an error occurs during a function call, Python prints the name of the function, and the name of the function that called it, and the name of the function that called that, all the way back to __main__.

For example, if you try to access cat from within print_twice, you get a NameError:

```
Traceback (innermost last):
  File "test.py", line 13, in __main__
    cat_and_print_twice(line1, line2)
  File "test.py", line 5, in cat_and_print_twice
    print_twice(cat)
  File "test.py", line 9, in print_twice
    print cat
NameError: name 'cat' is not defined
```

This list of functions is called a traceback. It tells you what program file the error occurred in, and what line, and what functions were executing at the time. It also shows the line of code that caused the error.

The order of the functions in the traceback is the same as the order of the frames in the stack diagram. The function that is currently running is at the bottom.

### 6.7 Fruitful functions and void functions

Some of the functions we are using, such as the math functions, yield results; for want of a better name, I call them **fruitful functions**. Other functions, like print_twice,
6.8. Why functions?

perform an action but don’t return a value. They are called **void functions**.

When you call a fruitful function, you almost always want to do something with the result; for example, you might assign it to a variable or use it as part of an expression:

\[
x = \text{math.cos(radians)} \\
golden = (\text{math.sqrt}(5) + 1) / 2
\]

When you call a function in interactive mode, Python displays the result:

```python
>>> math.sqrt(5)
2.2360679774997898
```

But in a script, if you call a fruitful function all by itself, the return value is lost forever!

```python
math.sqrt(5)
```

This script computes the square root of 5, but since it doesn’t store or display the result, it is not very useful.

Void functions might display something on the screen or have some other effect, but they don’t have a return value. If you try to assign the result to a variable, you get a special value called `None`.

```python
>>> result = print_twice('Bing')
Bing
Bing
```

```python
>>> print result
None
```

The value `None` is not the same as the string ‘None’. It is a special value that has its own type:

```python
>>> print type(None)
<type 'NoneType'>
```

The functions we have written so far are all void. We will start writing fruitful functions in a few sections.

### 6.8 Why functions?

It may not be clear why it is worth the trouble to divide a program into functions. There are a lot of reasons; here are a few:

- Creating a new function gives you an opportunity to name a group of statements, which makes your program easier to read and debug.
- Functions can make a program smaller by eliminating repetitive code. Later, if you make a change, you only have to make it in one place.
• Dividing a long program into functions allows you to debug the parts one at a
time and then assemble them into a working whole.

• Well-designed functions are often useful for many programs. Once you write
and debug one, you can reuse it.

6.9 Return values

Some of the built-in functions we have used, such as the math functions, produce re-
results. Calling the function generates a value, which we usually assign to a variable or
use as part of an expression.

e = math.exp(1.0)
height = radius * math.sin(radians)

All of the functions we have written so far are void; they print something or move
turtles around, but their return value is None.

In this chapter, we are (finally) going to write fruitful functions. The first example is
area, which returns the area of a circle with the given radius:

def area(radius):
    temp = math.pi * radius**2
    return temp

We have seen the return statement before, but in a fruitful function the return state-
ment includes a return value. This statement means: “Return immediately from this
function and use the following expression as a return value.” The expression provided
can be arbitrarily complicated, so we could have written this function more concisely:

def area(radius):
    return math.pi * radius**2

On the other hand, temporary variables like temp often make debugging easier.

Sometimes it is useful to have multiple return statements, one in each branch of a
conditional:

def absolute_value(x):
    if x < 0:
        return -x
    else:
        return x

Since these return statements are in an alternative conditional, only one will be exe-
cuted.
As soon as a return statement executes, the function terminates without executing any subsequent statements. Code that appears after a return statement, or any other place the flow of execution can never reach, is called **dead code**.

In a fruitful function, it is a good idea to ensure that every possible path through the program hits a return statement. For example:

```python
def absolute_value(x):
    if x < 0:
        return -x
    elif x > 0:
        return x
```

This program is not correct because if `x` happens to be 0, neither condition is true, and the function ends without hitting a return statement. If the flow of execution gets to the end of a function, the return value is None, which is not the absolute value of 0.

```python
>>> print absolute_value(0)
None
```

**Exercise 6.3.** Write a compare function that returns 1 if `x > y`, 0 if `x == y`, and -1 if `x < y`.

## 6.10 Boolean functions

Functions can return booleans, which is often convenient for hiding complicated tests inside functions. For example:

```python
def is_divisible(x, y):
    if x % y == 0:
        return True
    else:
        return False
```

It is common to give boolean functions names that sound like yes/no questions; `is_divisible` returns either `True` or `False` to indicate whether `x` is divisible by `y`.

Here is an example:

```python
>>> is_divisible(6, 4)
False
>>> is_divisible(6, 3)
True
```

The result of the `==` operator is a boolean, so we can write the function more concisely by returning it directly:

```python
def is_divisible(x, y):
    return x % y == 0
```
Boolean functions are often used in conditional statements:

```python
if is_divisible(x, y):
    print 'x is divisible by y'
```

It might be tempting to write something like:

```python
if is_divisible(x, y) == True:
    print 'x is divisible by y'
```

But the extra comparison is unnecessary.

**Exercise 6.4.** *Write a function* `is_between(x, y, z)` *that returns* `True` *if* $x \leq y \leq z$ *or* `False` *otherwise.*

### 6.11 Incremental development

As you write larger functions, you might start find yourself spending more time debugging.

To deal with increasingly complex programs, you might want to try a process called **incremental development.** The goal of incremental development is to avoid long debugging sessions by adding and testing only a small amount of code at a time.

As an example, suppose you want to find the distance between two points, given by the coordinates $(x_1, y_1)$ and $(x_2, y_2)$. By the Pythagorean theorem, the distance is:

\[
\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

The first step is to consider what a `distance` function should look like in Python. In other words, what are the inputs (parameters) and what is the output (return value)?

In this case, the two points are the inputs, which you can represent using four parameters. The return value is the distance, which is a floating-point value.

Already you can write an outline of the function:

```python
def distance(x1, y1, x2, y2):
    return 0.0
```

Obviously, this version doesn’t compute distances; it always returns zero. But it is syntactically correct, and it runs, which means that you can test it before you make it more complicated.

To test the new function, call it with sample arguments:

```python
>>> distance(1, 2, 4, 6)
0.0
```
6.11. Incremental development

I chose these values so that the horizontal distance is 3 and the vertical distance is 4; that way, the result is 5 (the hypotenuse of a 3-4-5 triangle). When testing a function, it is useful to know the right answer.

At this point we have confirmed that the function is syntactically correct, and we can start adding code to the body. A reasonable next step is to find the differences \( x_2 - x_1 \) and \( y_2 - y_1 \). The next version stores those values in temporary variables and prints them.

```python
def distance(x1, y1, x2, y2):
    dx = x2 - x1
    dy = y2 - y1
    print 'dx is', dx
    print 'dy is', dy
    return 0.0
```

If the function is working, it should display ‘dx is 3’ and ‘dy is 4’. If so, we know that the function is getting the right arguments and performing the first computation correctly. If not, there are only a few lines to check.

Next we compute the sum of squares of dx and dy:

```python
def distance(x1, y1, x2, y2):
    dx = x2 - x1
    dy = y2 - y1
    dsquared = dx**2 + dy**2
    print 'dsquared is: ', dsquared
    return 0.0
```

Again, you would run the program at this stage and check the output (which should be 25).

Finally, you can use `math.sqrt` to compute and return the result:

```python
def distance(x1, y1, x2, y2):
    dx = x2 - x1
    dy = y2 - y1
    dsquared = dx**2 + dy**2
    result = math.sqrt(dsquared)
    return result
```

If that works correctly, you are done. Otherwise, you might want to print the value of `result` before the return statement.

The final version of the function doesn’t display anything when it runs; it only returns a value. The print statements we wrote are useful for debugging, but once you get the function working, you should remove them. Code like that is called **scaffolding** because it is helpful for building the program but is not part of the final product.
When you start out, you should add only a line or two of code at a time. As you gain more experience, you might find yourself writing and debugging bigger chunks. Either way, incremental development can save you a lot of debugging time.

The key aspects of the process are:

1. Start with a working program and make small incremental changes. At any point, if there is an error, you should have a good idea where it is.

2. Use temporary variables to hold intermediate values so you can display and check them.

3. Once the program is working, you might want to remove some of the scaffolding or consolidate multiple statements into compound expressions, but only if it does not make the program difficult to read.

Exercise 6.5. Use incremental development to write a function called hypotenuse that returns the length of the hypotenuse of a right triangle given the lengths of the two legs as arguments. Record each stage of the development process as you go.

6.12 docstring

A docstring is a string at the beginning of a function that explains the interface (“doc” is short for “documentation”). Here is an example for our above distance function:

```python
def distance(x1, y1, x2, y2):
    # Calculates the distance between two points
    # when given their x and y numeric values.
    dx = x2 - x1
    dy = y2 - y1
    dsquared = dx**2 + dy**2
    result = math.sqrt(dsquared)
    return result
```

This docstring is a triple-quoted string, also known as a multi-line string because the triple quotes allow the string to span more than one line.

It is terse, but it contains the essential information someone would need to use this function. It explains concisely what the function does (without getting into the details of how it does it). It explains what effect each parameter has on the behavior of the function and what type each parameter should be (if it is not obvious).

Writing this kind of documentation is an important part of interface design. A well-designed interface should be simple to explain; if you are having a hard time explaining one of your functions, that might mean that the interface could be improved.
6.13 Composition

As you should expect by now, you can call one function from within another. This ability is called composition.

As an example, we’ll write a function that takes two points, the center of the circle and a point on the perimeter, and computes the area of the circle.

Assume that the center point is stored in the variables xc and yc, and the perimeter point is in xp and yp. The first step is to find the radius of the circle, which is the distance between the two points. Fortunately, there is a function, distance, that does that:

\[ \text{radius} = \text{distance}(xc, yc, xp, yp) \]

The next step is to find the area of a circle with that radius:

\[ \text{result} = \text{area}(\text{radius}) \]

Wrapping that up in a function, we get:

```python
def circle_area(xc, yc, xp, yp):
    radius = distance(xc, yc, xp, yp)
    result = area(radius)
    return result
```

The temporary variables radius and result are useful for development and debugging, but once the program is working, we can make it more concise by composing the function calls:

```python
def circle_area(xc, yc, xp, yp):
    return area(distance(xc, yc, xp, yp))
```

6.14 Debugging

As you start writing bigger programs, you might find yourself spending more time debugging. More code means more chances to make an error and more place for bugs to hide.

One way to cut your debugging time is “debugging by bisection.” For example, if there are 100 lines in your program and you check them one at a time, it would take 100 steps.

Instead, try to break the problem in half. Look at the middle of the program, or near it, for an intermediate value you can check. Add a print statement (or something else that has a verifiable effect) and run the program.

If the mid-point check is incorrect, the problem must be in the first half of the program. If it is correct, the problem is in the second half.
Every time you perform a check like this, you halve the number of lines you have to search. After six steps (which is much less than 100), you would be down to one or two lines of code.

At least in theory. In practice it is not always clear what the “middle of the program” is and not always possible to check it. It doesn’t make sense to count lines and find the exact midpoint. Instead, think about places in the program where there might be errors and places where it is easy to put a check. Then choose a spot where you think the chances are about the same that the bug is before or after the check.

6.15 Glossary

fruitful function: A function that returns a value.

void function: A function that doesn’t return a value.

function definition: A statement that creates a new function, specifying its name, parameters, and the statements it executes.

function object: A value created by a function definition. The name of the function is a variable that refers to a function object.

header: The first line of a function definition.

body: The sequence of statements inside a function definition.

parameter: A name used inside a function to refer to the value passed as an argument.

function call: A statement that executes a function. It consists of the function name followed by an argument list.

argument: A value provided to a function when the function is called. This value is assigned to the corresponding parameter in the function.

local variable: A variable defined inside a function. A local variable can only be used inside its function.

return value: The result of a function. If a function call is used as an expression, the return value is the value of the expression.

flow of execution: The order in which statements are executed during a program run.

stack diagram: A graphical representation of a stack of functions, their variables, and the values they refer to.

frame: A box in a stack diagram that represents a function call. It contains the local variables and parameters of the function.
6.16. Exercises

**traceback:** A list of the functions that are executing, printed when an exception occurs.

**temporary variable:** A variable used to store an intermediate value in a complex calculation.

**dead code:** Part of a program that can never be executed, often because it appears after a return statement.

**None:** A special value returned by functions that have no return statement or a return statement without an argument.

**incremental development:** A program development plan intended to avoid debugging by adding and testing only a small amount of code at a time.

**scaffolding:** Code that is used during program development but is not part of the final version.

**guardian:** A programming pattern that uses a conditional statement to check for and handle circumstances that might cause an error.

6.16 Exercises

**Exercise 6.6.** Fermat’s Last Theorem says that there are no integers \( a, b, \) and \( c \) such that

\[
a^n + b^n = c^n
\]

for any values of \( n \) greater than 2.

Write a function named `check_fermat` that takes four parameters—\( a, b, c \) and \( n \)—and that checks to see if Fermat’s theorem holds. If \( n \) is greater than 2 and it turns out to be true that

\[
a^n + b^n = c^n
\]

the program should print “Holy smokes, Fermat was wrong!” Otherwise the program should print “No, that doesn’t work.”

**Exercise 6.7.** Python provides a built-in function called `len` that returns the length of a string, so the value of `len('allen')` is 5.

Write a function named `right_justify` that takes a string named \( s \) as a parameter and that prints the string with enough leading spaces so that the last letter of the string is in column 70 of the display.

```python
>>> right_justify('allen')
allen
```
Exercise 6.8.

Write a function that draws grids like this in any size¹:

```
+ - - - - + - - - - +
|               |
|               |
+ - - - - + - - - - +
|               |
|               |
+ - - - - + - - - - +
```

Hint: to print more than one value on a line, you can print a comma-separated sequence:

```python
print '+', '-'
```

If the sequence ends with a comma, Python leaves the line unfinished, so the value printed next appears on the same line.

```python
print '+',
print '-'
```

The output of these statements is ‘+ -’.