How to Think Like a (Python 3.0) Programmer

Version 0.10.0

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Preface

The strange history of this book

In January 1999 I was preparing to teach an introductory programming class in Java. I had taught it three times and I was getting frustrated. The failure rate in the class was too high and even for students who succeeded, the overall level of achievement was too low.

One of the problems I saw was the books. I had tried three different books (and read a dozen more), and they all had the same problems. They were too big, with too much unnecessary detail about Java, and not enough high-level guidance about how to program. And they all suffered from the trap door effect: they would start out very gradual and easy, and then somewhere around Chapter 5, the bottom would fall out. The students would get too much new material, too fast, and I would spend the rest of the semester picking up the pieces.

Two weeks before the first day of classes, I decided to write my own book. I wrote one 10-page chapter a day for 13 days. I made some revisions on Day 14 and then sent it out to be photocopied.

My goals were:

- Keep it short. It is better for students to read 10 pages than not read 50 pages.
- Be careful with vocabulary. I tried to minimize the jargon and define each term at first use.
- Build gradually. To avoid trap doors, I took the most difficult topics and split them into a series of small steps.
- It's not about the language; it's about programming. I included the minumum useful subset of Java and left out the rest.

I needed a title, so on a whim I chose *How to Think Like a Computer Scientist*.

My first version was rough, but it worked. Students did the reading, and they understood enough that I could spend class time on the hard topics, the interesting topics and (most important) letting the students practice.

As a user and advocate of free software, I believe in the idea Benjamin Franklin expressed:

> "As we enjoy great Advantages from the Inventions of others, we should be glad of an Opportunity to serve others by any Invention of ours, and this we should do freely and generously."

So I released the book under the GNU Free Documenation License, which allows users to copy, modify, and distribute the book.

What happened next is the cool part. Jeff Elkner, a high school teacher in Virginia, adopted my book and translated it into Python. He sent me a copy of his translation, and I had the unusual experience of learning Python by reading my own book.

Jeff and I revised the book, incorporated a case study by Chris Meyers, and released *How to Think Like a Computer Scientist: Learning with Python*, also under the GNU Free Documenation License.

At the same time, my wife and I started Green Tea Press, which distributes several of my books electronically, and sells *How to Think* in hard copy.

I have been teaching with this book for more than five years now, and I have done a lot more Python programming. I still like the structure of the book, but for some time I have felt the need to make changes:

- Some of the examples in the first edition work better than others. In my classes I have discarded the less effective ones and developed improvements.
- There are only a few exercises in the first edition. Now I have five years of quizzes, exams and homeworks to choose from.
- I have been programming in Python for a while now and have a better appreciation of idiomatic Python. The book is still about programming, not Python, but now I think the book gets more leverage from the language.

At the same time, Jeff has been working on his own second edition, customized for his classes. Rather than cram everything into one book (which may be how other books got so big), we decided to work on different versions. They are both under the Free Documentation License, so users can choose one or combine material from both.

For my version, I am using the revised title *How to Think Like a (Python) Programmer*. This is a more modest goal than the original, but it might be more accurate.

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Needham MA

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I began my career teaching computer science in graduate school at the University of Wisconsin, using Java. While my students were learning, I felt there was too much syntax in the way of the core concepts of problem solving, and I found Java cumbersome to teach in any way except focusing on Objects First. After a few discussions with colleagues, I decided to move to Python, and began looking for textbook.

When I found Allen's text online, I first noticed the brevity and focus, along with constant reinforcement of good debugging techniques. And when I discovered that it was open source, I jumped at using it in my CS1 course. I have struggled with using texts before where the topic sequence does not match my preference, however, with this text I was free to edit and rearrange to suit my needs.

The text was very well received by my students, who appreciated the constant small inline examples, as well as the free price compared to textbooks for their other courses. As of this summer (2009) I am revising the text to use Python 3.0, and will be using this new version in the fall.

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Contributor List

To paraphrase the philosophy of the Free Software Foundation, this book is free like free speech, but not necessarily free like free pizza. It came about because of a collaboration that would not have been possible without the GNU Free Documentation License. So we thank the Free Software Foundation for developing this license and, of course, making it available to us.

We also thank the more than 100 sharp-eyed and thoughtful readers who have sent us suggestions and corrections over the past few years. In the spirit of free software, we decided to express our gratitude in the form of a contributor list. Unfortunately, this list is not complete, but we are doing our best to keep it up to date.

If you have a chance to look through the list, you should realize that each person here has spared you and all subsequent readers from the confusion of a technical error or a less-than-transparent explanation, just by sending us a note.

Impossible as it may seem after so many corrections, there may still be errors in this book. If you should stumble across one, please check the online version of the book at `http://thinkpython.com`, which is the most up-to-date version. If the error has not been corrected, please take a minute to send us email at `feedback@thinkpython.com`. If we make a change due to your suggestion, you will appear in the next version of the contributor list (unless you ask to be omitted). Thank you!

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Part I

Sequential Programming

Chapter 1

The way of the program

The goal of this book is to teach you to think like a computer scientist. This way of thinking combines some of the best features of mathematics, engineering, and natural science. Like mathematicians, computer scientists use formal languages to denote ideas (specifically computations). Like engineers, they design things, assembling components into systems and evaluating tradeoffs among alternatives. Like scientists, they observe the behavior of complex systems, form hypotheses, and test predictions.

The single most important skill for a computer scientist is **problem solving**. Problem solving means the ability to formulate problems, think creatively about solutions, and express a solution clearly and accurately. As it turns out, the process of learning to program is an excellent opportunity to practice problem-solving skills. That's why this chapter is called, "The way of the program."

On one level, you will be learning to program, a useful skill by itself. On another level, you will use programming as a means to an end. As we go along, that end will become clearer.

1.1 The Python programming language

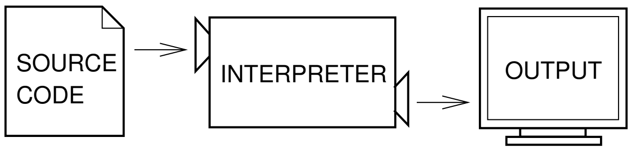
The programming language you will be learning is Python. Python is an example of a **high-level language**; other high-level languages you might have heard of are C, C++, Perl, and Java.

As you might infer from the name "high-level language," there are also **low-level languages**, sometimes referred to as "machine languages" or "assembly languages." Loosely speaking, computers can only execute programs written in low-level languages. So programs written in a high-level language have to be processed before they can run. This extra processing takes some time, which is a small disadvantage of high-level languages.

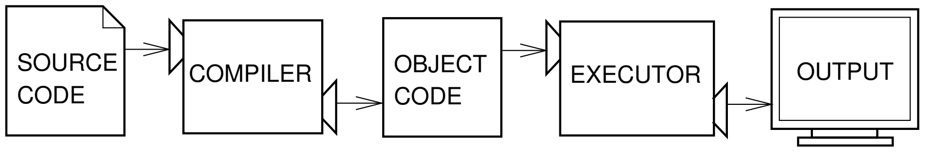
But the advantages are enormous. First, it is much easier to program in a high-level language. Programs written in a high-level language take less time to write, they are shorter and easier to read, and they are more likely to be correct. Second, high-level languages are **portable**, meaning that they can run on different kinds of computers with few or no modifications. Low-level programs can run on only one kind of computer and have to be rewritten to run on another.

Due to these advantages, almost all programs are written in high-level languages. Low-level languages are used only for a few specialized applications.

Two kinds of programs process high-level languages into low-level languages: **interpreters** and **compilers**. An interpreter reads a high-level program and executes it, meaning that it does what the program says. It processes the program a little at a time, alternately reading lines and performing computations.



A compiler reads the program and translates it completely before the program starts running. In this case, the high-level program is called the **source code**, and the translated program is called the **object code** or the **executable**. Once a program is compiled, you can execute it repeatedly without further translation.



Python is considered an interpreted language because Python programs are executed by an interpreter. There are two ways to use the interpreter: interactive mode and script mode. In interactive mode, you type Python programs and the interpreter prints the result:

```
Python 3.0.1+ (r301:69556, Apr 15 2009, 17:25:52)
Type "help", "copyright", "credits" or "license" for more information.
>>> print(1 + 1)
2
```

The first two lines in this example are displayed by the interpreter when it starts up. The third line starts with `>>>`, which is the **prompt** the interpreter uses to indicate that it is ready. If you type `print(1 + 1)`, the interpreter replies `2`.

Alternatively, you can store code in a file and use the interpreter to execute the contents of the file. Such a file is called a **script**. For example, you could use a text editor to create a file named `dinsdale.py` with the following contents:

```
print(1 + 1)
```

By convention, Python scripts have names that end with `.py`.

To execute the script, you have to tell the interpreter the name of the file. In a UNIX command window, you would type `python dinsdale.py`. In other development environments, the details of executing scripts are different.

Working in interactive mode is convenient for testing small pieces of code because you can type and execute them immediately. But for anything more than a few lines, you should save your code as a script so you can modify and execute it in the future.

1.2 What is a program?

A **program** is a sequence of instructions that specifies how to perform a computation. The computation might be something mathematical, such as solving a system of equations or finding the roots of a polynomial, but it can also be a symbolic computation, such as searching and replacing text in a document or (strangely enough) compiling a program.

The details look different in different languages, but a few basic instructions appear in just about every language:

input: Get data from the keyboard, a file, or some other device.

output: Display data on the screen or send data to a file or other device.

math: Perform basic mathematical operations like addition and multiplication.

conditional execution: Check for certain conditions and execute the appropriate sequence of statements.

repetition: Perform some action repeatedly, usually with some variation.

Believe it or not, that's pretty much all there is to it. Every program you've ever used, no matter how complicated, is made up of instructions that look pretty much like these. So you can think of programming as the process of breaking a large, complex task into smaller and smaller subtasks until the subtasks are simple enough to be performed with one of these basic instructions.

1.3 Algorithms

An **algorithm** is a mechanical process for solving a category of problems.

It is not easy to define an algorithm. It might help to start with something that is not an algorithm. When you learned to multiply single-digit numbers, you probably memorized the multiplication table. In effect, you memorized 100 specific solutions. That kind of knowledge is not algorithmic.

But if you were "lazy," you probably cheated by learning a few tricks. For example, to find the product of n and 9, you can write $n-1$ as the first digit and $10-n$ as the second digit. This trick is a general solution for multiplying any single-digit number by 9. That's an algorithm!

Similarly, the techniques you learned for addition with carrying, subtraction with borrowing, and long division are all algorithms. One of the characteristics of algorithms is that they do not require any intelligence to carry out. They are mechanical processes in which each step follows from the last according to a simple set of rules.

In my opinion, it is embarrassing that humans spend so much time in school learning to execute algorithms that, quite literally, require no intelligence.

On the other hand, the process of designing algorithms is interesting, intellectually challenging, and a central part of what we call programming.

Some of the things that people do naturally, without difficulty or conscious thought, are the hardest to express algorithmically. Understanding natural language is a good example. We all do it, but so far no one has been able to explain *how* we do it, at least not in the form of an algorithm.

1.4 What is debugging?

Programming is error-prone. For whimsical reasons, programming errors are called **bugs** and the process of tracking them down is called **debugging**.

Three kinds of errors can occur in a program: syntax errors, runtime errors, and semantic errors. It is useful to distinguish between them in order to track them down more quickly.

1.4.1 Syntax errors

Python can only execute a program if the syntax is correct; otherwise, the interpreter displays an error message. **Syntax** refers to the structure of a program and the rules about that structure. For example, in English, a sentence must begin with a capital letter and end with a period. this sentence contains a **syntax error**. So does this one

For most readers, a few syntax errors are not a significant problem, which is why we can read the poetry of e. e. cummings without spewing error messages. Python is not so forgiving. If there is a single syntax error anywhere in your program, Python will print an error message and quit, and you will not be able to run your program. During the first few weeks of your programming career, you will probably spend a lot of time tracking down syntax errors. As you gain experience, you will make fewer errors and find them faster.

1.4.2 Runtime errors

The second type of error is a runtime error, so called because the error does not appear until after the program has started running. These errors are also called **exceptions** because they usually indicate that something exceptional (and bad) has happened.

Runtime errors are rare in the simple programs you will see in the first few chapters, so it might be a while before you encounter one.

1.4.3 Semantic errors

The third type of error is the **semantic error**. If there is a semantic error in your program, it will run successfully, in the sense that the computer will not generate any error messages, but it will not do the right thing. It will do something else. Specifically, it will do what you told it to do.

The problem is that the program you wrote is not the program you wanted to write. The meaning of the program (its semantics) is wrong. Identifying semantic errors can be tricky because it requires you to work backward by looking at the output of the program and trying to figure out what it is doing.

1.4.4 Experimental debugging

One of the most important skills you will acquire is debugging. Although it can be frustrating, debugging is one of the most intellectually rich, challenging, and interesting parts of programming.

In some ways, debugging is like detective work. You are confronted with clues, and you have to infer the processes and events that led to the results you see.

Debugging is also like an experimental science. Once you have an idea about what is going wrong, you modify your program and try again. If your hypothesis was correct, then you can predict the result of the modification, and you take a step closer to a working program. If your hypothesis was wrong, you have to come up with a new one. As Sherlock Holmes pointed out, “When you have eliminated the impossible, whatever remains, however improbable, must be the truth.” (A. Conan Doyle, *The Sign of Four*)

For some people, programming and debugging are the same thing. That is, programming is the process of gradually debugging a program until it does what you want. The idea is that you should start with a program that does *something* and make small modifications, debugging them as you go, so that you always have a working program.

For example, Linux is an operating system that contains thousands of lines of code, but it started out as a simple program Linus Torvalds used to explore the Intel 80386 chip. According to Larry Greenfield, “One of Linus’s earlier projects was a program that would switch between printing AAAA and BBBB. This later evolved to Linux.” (*The Linux Users’ Guide* Beta Version 1)

Later chapters will make more suggestions about debugging and other programming practices.

1.5 Formal and natural languages

Natural languages are the languages people speak, such as English, Spanish, and French. They were not designed by people (although people try to impose some order on them); they evolved naturally.

Formal languages are languages that are designed by people for specific applications. For example, the notation that mathematicians use is a formal language that is particularly good at denoting relationships among numbers and symbols. Chemists use a formal language to represent the chemical structure of molecules. And most importantly:

> **Programming languages are formal languages that have been designed to express computations.**

Formal languages tend to have strict rules about syntax. For example, $3+3=6$ is a syntactically correct mathematical statement, but $3+=3\$6$ is not. H_2O is a syntactically correct chemical formula, but $_2Zz$ is not.

Syntax rules come in two flavors, pertaining to **tokens** and structure. Tokens are the basic elements of the language, such as words, numbers, and chemical elements. One of the problems with $3+=3\$6$ is that \$ is not a legal token in mathematics (at least as far as I know). Similarly, $_2Zz$ is not legal because there is no element with the abbreviation Zz.

The second type of syntax error pertains to the structure of a statement; that is, the way the tokens are arranged. The statement $3+=3\$6$ is illegal because even though $+$ and $=$ are legal tokens, you can't have one right after the other. Similarly, in a chemical formula the subscript comes after the element name, not before.
Exercise 1.1. *Write a well-structured English sentence with invalid tokens in it. Then write another sentence with all valid tokens but with invalid structure.*

When you read a sentence in English or a statement in a formal language, you have to figure out what the structure of the sentence is (although in a natural language you do this subconsciously). This process is called **parsing**.

For example, when you hear the sentence, "The penny dropped," you understand that "the penny" is the subject and "dropped" is the predicate. Once you have parsed a sentence, you can figure out what it means, or the semantics of the sentence. Assuming that you know what a penny is and what it means to drop, you will understand the general implication of this sentence.

Although formal and natural languages have many features in common—tokens, structure, syntax, and semantics—there are many differences:

ambiguity: Natural languages are full of ambiguity, which people deal with by using contextual clues and other information. Formal languages are designed to be nearly or completely unambiguous, which means that any statement has exactly one meaning, regardless of context.

redundancy: In order to make up for ambiguity and reduce misunderstandings, natural languages employ lots of redundancy. As a result, they are often verbose. Formal languages are less redundant and more concise.

literalness: Natural languages are full of idiom and metaphor. If I say, “The penny dropped,” there is probably no penny and nothing dropping[1]. Formal languages mean exactly what they say.

People who grow up speaking a natural language—everyone—often have a hard time adjusting to formal languages. In some ways, the difference between formal and natural language is like the difference between poetry and prose, but more so:

Poetry: Words are used for their sounds as well as for their meaning, and the whole poem together creates an effect or emotional response. Ambiguity is not only common but often deliberate.

Prose: The literal meaning of words is more important, and the structure contributes more meaning. Prose is more amenable to analysis than poetry but still often ambiguous.

Programs: The meaning of a computer program is unambiguous and literal, and can be understood entirely by analysis of the tokens and structure.

Here are some suggestions for reading programs (and other formal languages). First, remember that formal languages are much more dense than natural languages, so it takes longer to read them. Also, the structure is very important, so it is usually not a good idea to read from top to bottom, left to right. Instead, learn to parse the program in your head, identifying the tokens and interpreting the structure. Finally, the details matter. Small errors in spelling and punctuation, which you can get away with in natural languages, can make a big difference in a formal language.

1.6 The first program

Traditionally, the first program you write in a new language is called “Hello, World!” because all it does is display the words, “Hello, World!” In Python, it looks like this:

```
print('Hello, World!')
```

This is an example of the **print function**, which doesn’t actually print anything on paper. It displays a value on the screen. In this case, the result is the words

[1]This idiom means that someone realized something after a period of confusion.

```
Hello, World!
```

The quotation marks in the program mark the beginning and end of the text to be displayed; they don't appear in the result.

Some people judge the quality of a programming language by the simplicity of the "Hello, World!" program. By this standard, Python does about as well as possible.

1.7 Debugging

It is a good idea to read this book in front of a computer so you can try out the examples as you go. You can run most of the examples in interactive mode, but if you put the code into a script, it is easier to try out variations.

Whenever you are experimenting with a new feature, you should try to make mistakes. For example, in the "Hello, world!" program, what happens if you leave out one of the quotation marks? What if you leave out both? What if you leave out the parenthesis? What if you spell `print` wrong?

This kind of experiment helps you remember what you read; it also helps with debugging, because you get to know what the error messages mean. And that brings us to the First Theorem of Debugging:

> It is better to make mistakes now and on purpose than later and accidentally.

Learning to debug can be frustrating, but it is one of the most important parts of thinking like a computer scientist. At the end of each chapter there is a debugging section, like this one, with my thoughts (and theorems) of debugging. I hope they help!

1.8 Glossary

problem solving: The process of formulating a problem, finding a solution, and expressing the solution.

high-level language: A programming language like Python that is designed to be easy for humans to read and write.

low-level language: A programming language that is designed to be easy for a computer to execute; also called "machine language" or "assembly language."

portability: A property of a program that can run on more than one kind of computer.

interpret: To execute a program in a high-level language by translating it one line at a time.

compile: To translate a program written in a high-level language into a low-level language all at once, in preparation for later execution.

source code: A program in a high-level language before being compiled.

object code: The output of the compiler after it translates the program.

executable: Another name for object code that is ready to be executed.

prompt: Characters displayed by the interpreter to indicate that it is ready to take input from the user.

script: A program stored in a file (usually one that will be interpreted).

program: A set of instructions that specifies a computation.

algorithm: A general process for solving a category of problems.

bug: An error in a program.

debugging: The process of finding and removing any of the three kinds of programming errors.

syntax: The structure of a program.

syntax error: An error in a program that makes it impossible to parse (and therefore impossible to interpret).

exception: An error that is detected while the program is running.

semantics: The meaning of a program.

semantic error: An error in a program that makes it do something other than what the programmer intended.

natural language: Any one of the languages that people speak that evolved naturally.

formal language: Any one of the languages that people have designed for specific purposes, such as representing mathematical ideas or computer programs; all programming languages are formal languages.

token: One of the basic elements of the syntactic structure of a program, analogous to a word in a natural language.

parse: To examine a program and analyze the syntactic structure.

print function: A function that causes the Python interpreter to display a value on the screen.

1.9 Exercises

Exercise 1.2. *Use a web browser to go to* `http://python.org`*. This page contains a lot of information about Python, pointers to Python-related pages, and it gives you the ability to search the Python documentation.*

For example, if you enter `print` *in the search window, the first link that appears is the documentation of the* `print` *statement. At this point, not all of it will make sense to you, but it is good to know where it is!*

Exercise 1.3. *Start the Python interpreter and type* `help()` *to start the online help utility. Alternatively, you can type* `help(print)` *to get information about a particular topic, in this case the* `print` *statement. If this example doesn't work, you may need to install additional Python documentation or set an environment variable; unfortunately, the details depend on your operating system and version of Python.*

Chapter 2

Variables, expressions and statements

2.1 Values and classes

A **value** is one of the basic things a program works with, like a letter or a number. The values we have seen so far are `1`, `2`, and `'Hello, World!'`.

These values belong to different types of **classes**: `2` is an integer, and `'Hello, World!'` is a **string**, so-called because it contains a "string" of letters. You (and the interpreter) can identify strings because they are enclosed in quotation marks.

The print function also works for integers.

```
>>> print(4)
4
```

If you are not sure what class a value has, the interpreter can tell you.

```
>>> type('Hello, World!')
<class 'str'>
>>> type(17)
<class 'int'>
```

Not surprisingly, strings belong to the class `str` and integers belong to the class `int`. Less obviously, numbers with a decimal point belong to a class called `float`, because these numbers are represented in a format called **floating-point**.

```
>>> type(3.2)
<class 'float'>
```

What about values like `'17'` and `'3.2'`? They look like numbers, but they are in quotation marks like strings.

```
>>> type('17')
<class 'str'>
>>> type('3.2')
<class 'str'>
```

They're strings.

When you type a large integer, you might be tempted to use commas between groups of three digits, as in `1,000,000`. This is not a legal integer in Python, but it is legal:

```
>>> print(1,000,000)
1 0 0
```

Well, that's not what we expected at all! Python interprets `1,000,000` as a comma-separated sequence of integers which it prints with spaces between.

This is the first example we have seen of a semantic error: the code runs without producing an error message, but it doesn't do the "right" thing.

2.2 Variables

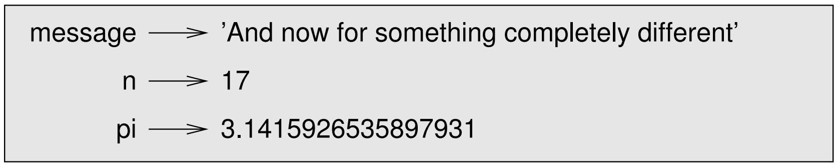
One of the most powerful features of a programming language is the ability to manipulate **variables**. A variable is a name that refers to a value.

The **assignment statement** creates new variables and gives them values:

```
>>> message = 'And now for something completely different'
>>> n = 17
>>> pi = 3.1415926535897931
```

This example makes three assignments. The first assigns a string to a new variable named `message`; the second gives the integer `17` to `n`; the third assigns the (approximate) value of π to `pi`.

A common way to represent variables on paper is to write the name with an arrow pointing to the variable's value. This kind of figure is called a **state diagram** because it shows what state each of the variables is in (think of it as the variable's state of mind). This diagram shows the result of the assignment statements:



The print function displays the value of a variable:

```
>>> print(n)
17
>>> print(pi)
3.14159265359
```

The type of a variable is the type of the value it refers to.

```
>>> type(message)
<class 'str'>
>>> type(n)
<class 'int'>
>>> type(pi)
<class 'float'>
```

2.3 Variable names and keywords

Programmers generally choose names for their variables that are meaningful—they document what the variable is used for.

Variable names can be arbitrarily long. They can contain both letters and numbers, but they have to begin with a letter. Although it is legal to use uppercase letters, by convention we don't. If you do, remember that case matters. `Bruce` and `bruce` are different variables.

The underscore character (_) can appear in a name. It is often used in names with multiple words, such as `my_name` or `airspeed_of_unladen_swallow`.

If you give a variable an illegal name, you get a syntax error:

```
>>> 76trombones = 'big parade'
SyntaxError: invalid syntax
>>> more@ = 1000000
SyntaxError: invalid syntax
>>> class = 'Advanced Theoretical Herpetology'
SyntaxError: invalid syntax
```

`76trombones` is illegal because it does not begin with a letter. `more@` is illegal because it contains an illegal character, `@`. But what's wrong with `class`?

It turns out that `class` is one of Python's **keywords**. The interpreter uses keywords to recognize the structure of the program, and they cannot be used as variable names.

Python has 31 keywords:

```
and       del       from      not       while
as        elif      global    or        with
assert    else      if        pass      yield
```

```
break     except    import    print
class     exec      in        raise
continue  finally   is        return
def       for       lambda    try
```

You might want to keep this list handy. If the interpreter complains about one of your variable names and you don't know why, see if it is on this list.

2.4 Statements

A statement is an instruction that the Python interpreter can execute. We have seen two kinds of statements: print and assignment.

When you type a statement on the command line, Python executes it and displays the result, if there is one.

A script usually contains a sequence of statements. If there is more than one statement, the results appear one at a time as the statements execute.

For example, the script

```
print(1)
x = 2
print(x)
```

produces the output

```
1
2
```

The assignment statement produces no output itself.

2.5 Operators and operands

Operators are special symbols that represent computations like addition and multiplication. The values the operator is applied to are called **operands**.

The following examples demonstrate the arithmetic operators:

```
20+32   hour-1   hour*60+minute   minute/60   5**2   (5+9)*(15-7)
```

The symbols `+`, `-`, and `/`, and the use of parenthesis for grouping, mean in Python what they mean in mathematics. The asterisk (`*`) is the symbol for multiplication, and `**` is the symbol for exponentiation.

When a variable name appears in the place of an operand, it is replaced with its value before the operation is performed.

Addition, subtraction, multiplication, division and exponentiation all do what you expect.

2.6 Floor Division and Modulus operator

The operators above have some interesting behavior in conjunction with integers.

```
>>> type(4 + 2)
<class 'int'>
>>> type(4 - 2)
<class 'int'>
>>> type(4 * 2)
<class 'int'>
>>> type(4 / 2)
<class 'float'>
```

Division will always return a floating-point number, even when the operands are integers. If we want an integer back from division, we will have to perform **floor division** with the symbol `//` . Floor division chops off the fraction part, so in this example it returns 2.

```
>>> 4 / 2
2.0
>>> 4 // 2
2
```

The **modulus operator** works on integers and yields the remainder when the first operand is divided by the second. In Python, the modulus operator is a percent sign (`%`). The syntax is the same as for other operators:

```
>>> quotient = 7 // 3
>>> print(quotient)
2
>>> remainder = 7 % 3
>>> print(remainder)
1
```

So 7 divided by 3 is 2 with 1 left over.

The modulus operator turns out to be surprisingly useful. For example, you can check whether one number is divisible by another—if `x % y` is zero, then `x` is divisible by `y`.

Also, you can extract the right-most digit or digits from a number. For example, `x % 10` yields the right-most digit of `x` (in base 10). Similarly `x % 100` yields the last two digits.

2.7 Expressions

An **expression** is a combination of values, variables, and operators. If you type an expression on the command line, the interpreter **evaluates** it and displays the result:

```
>>> 1 + 1
2
```

Although expressions can contain values, variables, and operators, not every expression contains all of these elements. A value all by itself is considered an expression, and so is a variable.

```
>>> 17
17
>>> x
2
```

In a script, an expression all by itself is a legal statement, but it doesn't do anything. The following script produces no output at all:

```
17
3.2
'Hello, World!'
1 + 1
```

If you want the script to display the values of these expressions, you have to use `print` statements.

2.8 Order of operations

When more than one operator appears in an expression, the order of evaluation depends on the **rules of precedence**. For mathematical operators, Python follows the mathematical rules. The acronym **PEMDAS** is a useful way to remember them:

- **P**arentheses have the highest precedence and can be used to force an expression to evaluate in the order you want. Since expressions in parentheses are evaluated first, `2 * (3-1)` is 4, and `(1+1)**(5-2)` is 8. You can also use parentheses to make an expression easier to read, as in `(minute * 100) / 60`, even though it doesn't change the result.

- **E**xponentiation has the next highest precedence, so `2**1+1` is 3 and not 4, and `3*1**3` is 3 and not 27.

- **M**ultiplication and **D**ivision (including Modulus) have the same precedence, which is higher than **A**ddition and **S**ubtraction, which also have the same precedence. So `2*3-1` is 5, not 4, and `6+4/2` is 8, not 5.

- Operators with the same precedence are evaluated from left to right. So in the expression `degrees / 2 * pi`, the division happens first and the result is multiplied by `pi`. If you meant to divide by 2π, you should have used parentheses.

2.9 Comments

As programs get bigger and more complicated, they get more difficult to read. Formal languages are dense, and it is often difficult to look at a piece of code and figure out what it is doing, or why.

For this reason, it is a good idea to add notes to your programs to explain in natural language what the program is doing. These notes are called **comments**, and they are marked with the # symbol:

```
# compute the percentage of the hour that has elapsed
percentage = (minute * 100) / 60
```

In this case, the comment appears on a line by itself. You can also put comments at the end of a line:

```
percentage = (minute * 100) / 60     # percentage of an hour
```

Everything from the # to the end of the line is ignored—it has no effect on the program.

Comments are most useful when they document non-obvious features of the code. It is reasonable to assume that the reader can figure out *what* the code does; it is much more useful to explain *why*.

This comment is redundant with the code and useless:

```
v = 5     # assign 5 to v
```

This comment contains useful information that is not in the code:

```
v = 5     # velocity in meters/second.
```

Good variable names can reduce the need for comments, but long names can make complex expressions hard to read, so there is a tradeoff.

2.10 Debugging

At this point the syntax error you are most likely to make is an illegal variable name, like `class` and `yield` (which are keywords) or `odd~job` and `US$` which contain illegal characters.

If you put a space in a variable name, Python thinks it is two operands without an operator:

```
>>> bad name = 5
SyntaxError: invalid syntax
```

For syntax errors, the error messages don't help much. The most common messages are `SyntaxError: invalid syntax` and `SyntaxError: invalid token`, neither of which is very informative.

The run-time error you are most likely to make is a "use before def;" that is, trying to use a variable before you have assigned a value. This can happen if you spell a variable name wrong:

```
>>> principal = 327.68
>>> interest = principle * rate
NameError: name 'principle' is not defined
```

Variables names are case sensitive, so `Bob` is not the same as `bob`.

At this point the most likely cause of a semantic error is the order of operations. For example, to evaluate $\frac{1}{2a}$, you might be tempted to write

```
>>> 1.0 / 2.0 * a
```

But the division happens first, so you would get $a/2$, which is not the same thing! Unfortunately, there is no way for Python to know what you intended to write, so in this case you don't get an error message; you just get the wrong answer.

And that brings us to the Second Theorem of Debugging:

> The only thing worse than getting an error message is not getting an error message.

2.11 Glossary

value: One of the basic units of data, like a number or string, that a program manipulates.

type: A function that tells us the category of a value. The classes we have seen so far are integers (class `int`), floating-point numbers (class `float`), and strings (class `str`).

integer: A class that represents whole numbers.

floating-point: A class that represents numbers with fractional parts.

string: A class that represents sequences of characters.

variable: A name that refers to a value.

statement: A section of code that represents a command or action. So far, the statements we have seen are assignments.

assignment: A statement that assigns a value to a variable.

state diagram: A graphical representation of a set of variables and the values they refer to.

keyword: A reserved word that is used by the compiler to parse a program; you cannot use keywords like `if`, `def`, and `while` as variable names.

operator: A special symbol that represents a simple computation like addition, multiplication, or string concatenation.

operand: One of the values on which an operator operates.

floor division: The operation that divides two numbers and chops off the fraction part.

modulus operator: An operator, denoted with a percent sign (`%`), that works on integers and yields the remainder when one number is divided by another.

expression: A combination of variables, operators, and values that represents a single result value.

evaluate: To simplify an expression by performing the operations in order to yield a single value.

rules of precedence: The set of rules governing the order in which expressions involving multiple operators and operands are evaluated.

comment: Information in a program that is meant for other programmers (or anyone reading the source code) and has no effect on the execution of the program.

2.12 Exercises

Exercise 2.1. *Assume that we execute the following assignment statements:*

```
width = 17
height = 12.0
delimiter = '.'
```

For each of the following expressions, write the value of the expression and the class (of the value of the expression).

1. `width/2`
2. `height/3.0`
3. `width/2.0`
4. `1 + 2 * 5`
5. `delimiter * 5`

Exercise 2.2. *Practice using the Python interpreter as a calculator:*

- *If you ran 10 kilometers in 45 minutes 30 seconds, what was your average pace in minutes per mile? What was your average speed in miles per hour? (Hint: there are 1.61 kilometers in a mile).*

Chapter 3

Using Functions

3.1 Function calls

In the context of programming, a **function** is a named sequence of statements that performs a computation. When you define a function, you specify the name and the sequence of statements. Later, you can “call” the function by name. We have already seen one example of a **function call**:

```
>>> type('32')
<class 'str'>
```

The name of the function is `type`. The expression in parentheses is called the **argument** of the function. The result, for this function, is the class of the argument, which is a string.

It is common to say that a function “takes” an argument and “returns” a result. The result is called the **return value**.

When you call a function in interactive mode, the interpreter displays the return value, but in a script a function call, all by itself, doesn’t display anything. To see the result, you have to print it:

```
print(type('32'))
```

Or assign the return value to a variable, which you can print (or use for some other purpose) later.

```
stereo = type('32')
print(stereo)
```

3.2 Type conversion functions

Python provides built-in functions that convert values from one class to another. The `int` function takes any value and converts it to an integer if it can or complains otherwise:

```
>>> int('32')
32
>>> int('Hello')
ValueError: invalid literal for int(): Hello
```

`int` can convert floating-point values to integers, but it doesn't round off; it chops off the fraction part:

```
>>> int(3.99999)
3
>>> int(-2.3)
-2
```

`float` converts integers and strings to floating-point numbers:

```
>>> float(32)
32.0
>>> float('3.14159')
3.14159
```

Finally, `str` converts its argument to a string:

```
>>> str(32)
'32'
>>> str(3.14149)
'3.14149'
```

3.3 Keyboard input

The programs we have written so far are a bit rude in the sense that they accept no input from the user. They just do the same thing every time.

Python provides a built-in function called `input` that gets input from the keyboard. When this function is called, the program stops and waits for the user to type something. When the user presses Return or Enter, the program resumes and `input` returns what the user typed as a string.

```
>>> inp = input()
What are you waiting for?
>>> print(input)
What are you waiting for?
```

Before calling `input`, it is a good idea to print a prompt telling the user what to input. `input` takes a prompt as an argument:

```
>>> name = input('What...is your name?\n')
What...is your name?
Arthur, King of the Britons!
>>> print(name)
Arthur, King of the Britons!
```

The sequence `\n` at the end of the prompt represents a newline, which is a special character that causes a line break. That's why the user's input appears below the prompt.

If you expect the user to type an integer, you can try to convert the return value to `int`:

```
>>> prompt = 'What...is the airspeed velocity of an unladen swallow?\n'
>>> speed = input(prompt)
What...is the airspeed velocity of an unladen swallow?
17
>>> int(speed)
17
```

But if the user types something other than a string of digits, you get an exception:

```
>>> speed = input(prompt)
What...is the airspeed velocity of an unladen swallow?
What do you mean, an African or a European swallow?
>>> int(speed)
ValueError: invalid literal for int()
```

We will see how to handle this kind of error later.

3.4 Math functions

Python has a math module that provides most of the familiar mathematical functions. A **module** is a file that contains a collection of related functions.

Before we can use the module, we have to import it:

```
>>> import math
```

This statement creates a **module object** named math. If you print the module object, you get some information about it:

```
>>> print(math)
<module 'math' from '/usr/lib/python3.0/lib-dynload/mathmodule.so'>
```

The module object contains the functions and variables defined in the module. To access one of the functions, you have to specify the name of the module and the name of the function, separated by a dot (also known as a period). This format is called **dot notation**.

```
>>> ratio = signal_power / noise_power
>>> decibels = 10 * math.log10(ratio)

>>> radians = 0.7
>>> height = math.sin(radians)
```

The first example computes the logarithm base 10 of the signal-to-noise ratio. The math module also provides a function called `log` that computes logarithms base `e`.

The second example finds the sine of `radians`. The name of the variable is a hint that `sin` and the other trigonometric functions (`cos`, `tan`, etc.) take arguments in radians. To convert from degrees to radians, divide by 360 and multiply by 2π:

```
>>> degrees = 45
>>> radians = degrees / 360.0 * 2 * math.pi
>>> math.sin(radians)
0.707106781187
```

The expression `math.pi` gets the variable `pi` from the math module. Conveniently, the value of this variable is an approximation of π, accurate to about 15 digits.

If you know your trigonometry, you can check the previous result by comparing it to the square root of two divided by two:

```
>>> math.sqrt(2) / 2.0
0.707106781187
```

3.5 Composition

So far, we have looked at the elements of a program—variables, expressions, and statements—in isolation, without talking about how to combine them.

One of the most useful features of programming languages is their ability to take small building blocks and **compose** them. For example, the argument of a function can be any kind of expression, including arithmetic operators:

```
x = math.sin(degrees / 360.0 * 2 * math.pi)
```

And even function calls:

```
x = math.exp(math.log(x+1))
```

Almost anywhere you can put a value, you can put an arbitrary expression, with one exception: the left side of an assignment statement has to be a variable name. An expression on the left side is a syntax error.

```
>>> minutes = hours * 60                 # right
>>> hours * 60 = minutes                 # wrong!
SyntaxError: can't assign to operator
```

3.6 Debugging

If you are using a text editor to write your scripts, you might run into problems with spaces and tabs. The best way to avoid these problems is to use spaces exclusively (no tabs). Most text editors that know about Python do this by default, but some don't.

Tabs and spaces are usually invisible, which makes them hard to debug, so try to find an editor that manages indentation for you.

Also, don't forget to save your program before you run it. Some development environments do this automatically, but some don't. In that case the program you are looking at in the text editor is not the same as the program you are running (the one on disk).

Debugging can take a long time if you keep running the same, incorrect, program over and over! And that brings me to the Third Theorem of Debugging:

> Make sure that the code you are looking at is the code you are running.

If you're not sure, put something like `print('hello!')` at the beginning of the program and run it again. If you don't see `'hello!'`, you're not running the right program!

3.7 Glossary

function: A named sequence of statements that performs some useful operation. Functions may or may not take arguments and may or may not produce a result.

module: A file that contains a collection of related functions and other definitions.

import statement: A statement that reads a module file and creates a module object.

module object: A value created by an `import` statement that provides access to the values defined in a module.

dot notation: The syntax for calling a function in another module by specifying the module name followed by a dot (period) and the function name.

composition: Using an expression as part of a larger expression, or a statement as part of a larger statement.

3.8 Exercises

Part II

Decisions, Detours and Data Structures

Chapter 4

Conditionals

4.1 Boolean expressions

A **boolean expression** is an expression that is either true or false. The following examples use the operator ==, which compares two operands and produces `True` if they are equal and `False` otherwise:

```
>>> 5 == 5
True
>>> 5 == 6
False
```

`True` and `False` are special values that belong to the class `bool`; they are not strings:

```
>>> type(True)
<class 'bool'>
>>> type(False)
<class 'bool'>
```

The == operator is one of the **comparison operators**; the others are:

```
      x != y               # x is not equal to y
      x > y                # x is greater than y
      x < y                # x is less than y
      x >= y               # x is greater than or equal to y
      x <= y               # x is less than or equal to y
```

Although these operations are probably familiar to you, the Python symbols are different from the mathematical symbols. A common error is to use a single equal sign (=) instead of a double equal sign (==). Remember that = is an assignment operator and == is a comparison operator. There is no such thing as =< or =>.

4.2 Logical operators

There are three **logical operators**: `and`, `or`, and `not`. The semantics (meaning) of these operators is similar to their meaning in English. For example, `x > 0 and x < 10` is true only if `x` is greater than 0 *and* less than 10.

`n%2 == 0 or n%3 == 0` is true if *either* of the conditions is true, that is, if the number is divisible by 2 *or* 3.

Finally, the `not` operator negates a boolean expression, so `not (x > y)` is true if `x > y` is false, that is, if `x` is less than or equal to `y`.

Strictly speaking, the operands of the logical operators should be boolean expressions, but Python is not very strict. Any nonzero number is interpreted as "True."

```
>>> 17 and True
True
```

This flexibility can be useful, but there are some subtleties to it that might be confusing. You might want to avoid it (unless you know what you are doing).

4.3 Conditional execution

In order to write useful programs, we almost always need the ability to check conditions and change the behavior of the program accordingly. **Conditional statements** give us this ability. The simplest form is the `if` statement:

```
if x > 0:
    print('x is positive')
```

The boolean expression after the `if` statement is called the **condition**. If it is true, then the indented statement gets executed. If not, nothing happens.

`if` statements have a header followed by an indented block. Statements like this are called **compound statements**.

There is no limit on the number of statements that can appear in the body, but there has to be at least one. Occasionally, it is useful to have a body with no statements (usually as a place keeper for code you haven't written yet). In that case, you can use the `pass` statement, which does nothing.

```
if x < 0:
    pass          # need to handle negative values!
```

4.4 Alternative execution

A second form of the `if` statement is alternative execution, in which there are two possibilities and the condition determines which one gets executed. The syntax looks like this:

```
if x%2 == 0:
    print('x is even')
else:
    print('x is odd')
```

If the remainder when `x` is divided by 2 is 0, then we know that `x` is even, and the program displays a message to that effect. If the condition is false, the second set of statements is executed. Since the condition must be true or false, exactly one of the alternatives will be executed. The alternatives are called **branches**, because they are branches in the flow of execution.

4.5 Chained conditionals

Sometimes there are more than two possibilities and we need more than two branches. One way to express a computation like that is a **chained conditional**:

```
if x < y:
    print('x is less than y')
elif x > y:
    print('x is greater than y')
else:
    print('x and y are equal')
```

`elif` is an abbreviation of "else if." Again, exactly one branch will be executed. There is no limit on the number of `elif` statements. If there is an `else` clause, it has to be at the end, but there doesn't have to be one.

```
if choice == 1:
    function1()
elif choice == 2:
    function2()
elif choice == 3:
    function3()
```

Each condition is checked in order. If the first is false, the next is checked, and so on. If one of them is true, the corresponding branch executes, and the statement ends. Even if more than one condition is true, only the first true branch executes.

4.6 Nested conditionals

One conditional can also be nested within another. We could have written the trichotomy example like this:

```
if x == y:
    print('x and y are equal')
else:
    if x < y:
        print('x is less than y')
    else:
        print('x is greater than y')
```

The outer conditional contains two branches. The first branch contains a simple statement. The second branch contains another `if` statement, which has two branches of its own. Those two branches are both simple statements, although they could have been conditional statements as well.

Although the indentation of the statements makes the structure apparent, nested conditionals become difficult to read very quickly. In general, it is a good idea to avoid them when you can.

Logical operators often provide a way to simplify nested conditional statements. For example, we can rewrite the following code using a single conditional:

```
if 0 < x:
    if x < 10:
        print('x is a positive single digit.')
```

The `print` statement is executed only if we make it past both conditionals, so we can get the same effect with the `and` operator:

```
if 0 < x and x < 10:
    print('x is a positive single digit.')
```

4.7 String comparison

The comparison operators work on strings. To see if two strings are equal:

```
if word == 'banana':
    print('Yes, we have no bananas!')
```

Other comparison operations are useful for putting words in alphabetical order:

```
if word < 'banana':
    print('Your word,' + word + ', comes before banana.')
elif word > 'banana':
    print('Your word,' + word + ', comes after banana.')
else:
    print('Yes, we have no bananas!')
```

Python does not handle uppercase and lowercase letters the same way that people do. All the uppercase letters come before all the lowercase letters, so:

```
Your word, Zebra, comes before banana.
```

A common way to address this problem is to convert strings to a standard format, such as all lowercase, before performing the comparison. The more difficult problem is making the program realize that zebras are not fruit.

4.8 Random numbers

Most computer programs do the same thing every time they execute, given the same inputs, so they are said to be **deterministic**. Determinism is usually a good thing, since we expect the same calculation to yield the same result. For some applications, though, we want the computer to be unpredictable. Games are an obvious example, but there are more.

Making a program truly nondeterministic turns out to be not so easy, but there are ways to make it at least seem nondeterministic. One of them is to use algorithms that generate **pseudorandom** numbers. Pseudorandom numbers are not truly random because they are generated by a deterministic computation, but just by looking at the numbers it is all but impossible to distinguish them from random.

The `random` module provides functions that generate pseudorandom numbers (which I will simply call "random" from here on).

The function `random` returns a random float between 0.0 and 1.0 (including 0.0 but not 1.0). Each time you call `random`, you get the next number in a long series. We can use this to simulate flipping a coin with a 50% probability of Heads and a 50% probability of Tails:

```
import random

x = random.random()
if x > 0.5:
    print('Heads')
else:
    print('Tails')
```

The function `randint` takes parameters `low` and `high` and returns an integer between `low` and `high` (including both).

```
>>> random.randint(5, 10)
5
>>> random.randint(5, 10)
9
```

The `random` module also provides functions to generate random values from continuous distributions including Gaussian, exponential, gamma, and a few more.

4.9 Debugging

The traceback Python displays when an error occurs contains a lot of information, but it can be overwhelming, especially when there are many frames on the stack. The most useful pieces are usually:

- what kind of error it was, and
- where it occurred.

Syntax errors are usually easy to find, but there are a few gotchas. Whitespace errors can be tricky because spaces and tabs are invisible and we are used to ignoring them.

```
>>> x = 5
>>>  y = 6
  File "<stdin>", line 1
    y = 6
    ^
SyntaxError: invalid syntax
```

In this example, the problem is that the second line is indented by one space. But the error message points to `y`, which is misleading. In general, error messages indicate where the error was discovered, but the actual error might be earlier in the code, sometimes on a previous line.

And that brings me to the Fourth Theorem of Debugging:

> Error messages tell you where the problem was discovered, but that is often not where it was caused.

4.10 Glossary

boolean expression: An expression whose value is either `True` or `False`.

comparison operator: One of the operators that compares its operands: `==`, `!=`, `>`, `<`, `>=`, and `<=`.

logical operator: One of the operators that combines boolean expressions: `and`, `or`, and `not`.

conditional statement: A statement that controls the flow of execution depending on some condition.

condition: The boolean expression in a conditional statement that determines which branch is executed.

compound statement: A statement that consists of a header and a body. The header ends with a colon (:). The body is indented relative to the header.

body: The sequence of statements within a compound statement.

branch: One of the alternative sequences of statements in a conditional statement.

chained conditional: A conditional statement with a series of alternative branches.

4.11 Exercises

Chapter 5

Writing functions

5.1 Adding new functions

So far, we have only been using the functions that come with Python, but it is also possible to add new functions. A **function definition** specifies the name of a new function and the sequence of statements that execute when the function is called.

Here is an example:

```
def print_lyrics():
    print("I'm a lumberjack, and I'm okay.")
    print("I sleep all night and I work all day.")
```

`def` is a keyword that indicates that this is a function definition. The name of the function is `print_lyrics`. The rules for function names are the same as for variable names: letters, numbers and some punctuation marks are legal, but the first character can't be a number. You can't use a keyword as the name of a function, and you should avoid having a variable and a function with the same name.

The empty parentheses after the name indicate that this function doesn't take any arguments.

The first line of the function definition is called the **header**; the rest is called the **body**. The header has to end with a colon and the body has to be indented. By convention, the indentation is always four spaces. The body can contain any number of statements.

The strings in the print functions are enclosed in double quotes. Single quotes and double quotes do the same thing. Most people use single quotes except in cases like this where a single quote (which is also an apostrophe) appears in the string.

If you type a function definition in interactive mode, the interpreter prints ellipses (...) to let you know that the definition isn't complete:

```
>>> def print_lyrics():
...     print("I'm a lumberjack, and I'm okay.")
...     print("I sleep all night and I work all day.")
...
```

To end the function, you have to enter an empty line (this is not necessary in a script).

Defining a function creates a variable with the same name.

```
>>> print(print_lyrics)
<function print_lyrics at 0xb7e99e9c>
>>> print(type(print_lyrics))
<class 'function'>
```

The value of `print_lyrics` is a **function object**, which has class `function`.

The syntax for calling the new function is the same as for built-in functions:

```
>>> print_lyrics()
I'm a lumberjack, and I'm okay.
I sleep all night and I work all day.
```

Once you have defined a function, you can use it inside another function. For example, to repeat the previous refrain, we could write a function called `repeat_lyrics`:

```
def repeat_lyrics():
    print_lyrics()
    print_lyrics()
```

And then call `repeat_lyrics`:

```
>>> repeat_lyrics()
I'm a lumberjack, and I'm okay.
I sleep all night and I work all day.
I'm a lumberjack, and I'm okay.
I sleep all night and I work all day.
```

But that’s not really how the song goes.

5.2 Definitions and uses

Pulling together the code fragments from the previous section, the whole program looks like this:

```
def print_lyrics():
    print("I'm a lumberjack, and I'm okay.")
    print("I sleep all night and I work all day.")
```

```
def repeat_lyrics():
    print_lyrics()
    print_lyrics()

repeat_lyrics()
```

This program contains two function definitions: `print_lyrics` and `repeat_lyrics`. Function definitions get executed just like other statements, but the effect is to create the new function. The statements inside the function do not get executed until the function is called, and the function definition generates no output.

As you might expect, you have to create a function before you can execute it. In other words, the function definition has to be executed before the first time it is called.
Exercise 5.1. *Move the last line of this program to the top, so the function call appears before the definitions. Run the program and see what error message you get.*
Exercise 5.2. *Move the function call back to the bottom and move the definition of* `print_lyrics` *after the definition of* `repeat_lyrics`*. What happens when you run this program?*

5.3 Flow of execution

In order to ensure that a function is defined before its first use, you have to know the order in which statements are executed, which is called the **flow of execution**.

Execution always begins at the first statement of the program. Statements are executed one at a time, in order from top to bottom.

Function definitions do not alter the flow of execution of the program, but remember that statements inside the function are not executed until the function is called.

A function call is like a detour in the flow of execution. Instead of going to the next statement, the flow jumps to the body of the function, executes all the statements there, and then comes back to pick up where it left off.

That sounds simple enough, until you remember that one function can call another. While in the middle of one function, the program might have to execute the statements in another function. But while executing that new function, the program might have to execute yet another function!

Fortunately, Python is adept at keeping track of where it is, so each time a function completes, the program picks up where it left off in the function that called it. When it gets to the end of the program, it terminates.

What's the moral of this sordid tale? When you read a program, you don't always want to read from top to bottom. Sometimes it makes more sense if you follow the flow of execution.

5.4 Why functions?

It may not be clear why it is worth the trouble to divide a program into functions. There are a lot of reasons; here are a few:

- Creating a new function gives you an opportunity to name a group of statements, which makes your program easier to read and debug.
- Functions can make a program smaller by eliminating repetitive code. Later, if you make a change, you only have to make it in one place.
- Dividing a long program into functions allows you to debug the parts one at a time and then assemble them into a working whole.
- Well-designed functions are often useful for many programs. Once you write and debug one, you can reuse it.

5.5 Parameters and arguments

Some of the built-in functions you have used require arguments. For example, when you call `math.sin` you pass a number (in radians) as an argument. Some functions take more than one argument; `math.pow` takes two, the base and the exponent.

Inside the function, the arguments are assigned to variables called **parameters**. Here is an example of a user-defined function that takes an argument:

```
def print_twice(bruce):
    print(bruce)
    print(bruce)
```

This function assigns the argument to a parameter named `bruce`. When the function is called, it prints the value of the parameter, whatever it is, twice.

This function works with any value that can be printed.

```
>>> print_twice('Spam')
Spam
Spam
>>> print_twice(17)
17
17
>>> print_twice(math.pi)
3.14159265359
3.14159265359
```

The same rules of composition that apply to built-in functions also apply to user-defined functions, so we can use any kind of expression as an argument for `print_twice`:

```
>>> print_twice('Spam '*4)
Spam Spam Spam Spam
Spam Spam Spam Spam
>>> print_twice(math.cos(math.pi))
-1.0
-1.0
```

The argument is evaluated before the function is called, so in the examples the expressions `'Spam '*4` and `math.cos(math.pi)` are only evaluated once.

You can also use a variable as an argument:

```
>>> michael = 'Eric, the half a bee.'
>>> print_twice(michael)
Eric, the half a bee.
Eric, the half a bee.
```

The name of the variable we pass as an argument (`michael`) has nothing to do with the name of the parameter (`bruce`). It doesn't matter what the value was called back home (in the caller); here in `print_twice`, we call everybody `bruce`.

5.6 Variables and parameters are local

When you create a variable inside a function, it is **local**, which means that it only exists inside the function. For example:

```
def cat_twice(part1, part2):
    cat = part1 + part2
    print_twice(cat)
```

This function takes two arguments, concatenates them, and prints the result twice. Here is an example that uses it:

```
>>> line1 = 'Bing tiddle '
>>> line2 = 'tiddle bang.'
>>> cat_twice(line1, line2)
Bing tiddle tiddle bang.
Bing tiddle tiddle bang.
```

When `cat_twice` terminates, the variable `cat` is destroyed. If we try to print it, we get an exception:

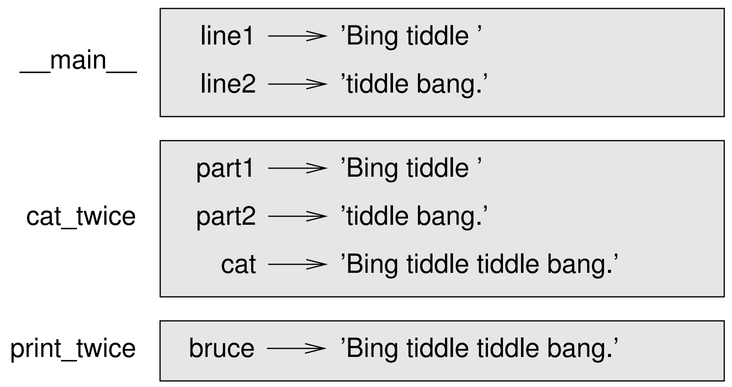
```
>>> print(cat)
NameError: name 'cat' is not defined
```

Parameters are also local. For example, outside `print_twice`, there is no such thing as `bruce`.

5.7 Stack diagrams

To keep track of which variables can be used where, it is sometimes useful to draw a **stack diagram**. Like state diagrams, stack diagrams show the value of each variable, but they also show the function each variable belongs to.

Each function is represented by a **frame**. A frame is a box with the name of a function beside it and the parameters and variables of the function inside it. The stack diagram for the previous example looks like this:



The frames are arranged in a stack that indicates which function called which, and so on. In this example, `print_twice` was called by `cat_twice`, and `cat_twice` was called by `__main__`, which is a special name for the topmost frame. When you create a variable outside of any function, it belongs to `__main__`.

Each parameter refers to the same value as its corresponding argument. So, `part1` has the same value as `line1`, `part2` has the same value as `line2`, and `bruce` has the same value as `cat`.

If an error occurs during a function call, Python prints the name of the function, and the name of the function that called it, and the name of the function that called *that*, all the way back to `__main__`.

For example, if you try to access `cat` from within `print_twice`, you get a `NameError`:

```
Traceback (innermost last):
  File "test.py", line 13, in __main__
    cat_and_print_twice(line1, line2)
  File "test.py", line 5, in cat_and_print_twice
    print_twice(cat)
  File "test.py", line 9, in print_twice
    print(cat)
NameError: name 'cat' is not defined
```

This list of functions is called a **traceback**. It tells you what program file the error occurred in, and what line, and what functions were executing at the time. It also shows the line of code that caused the error.

The order of the functions in the traceback is the same as the order of the frames in the stack diagram. The function that is currently running is at the bottom.

5.8 Fruitful functions and void functions

Some of the functions we are using, such as the math functions, yield results; for want of a better name, I call them **fruitful functions**. Other functions, like `print_twice`, perform an action but don't return a value. They are called **void functions**.

When you call a fruitful function, you almost always want to do something with the result; for example, you might assign it to a variable or use it as part of an expression:

```
x = math.cos(radians)
golden = (math.sqrt(5) + 1) / 2
```

When you call a function in interactive mode, Python displays the result:

```
>>> math.sqrt(5)
2.2360679774997898
```

But in a script, if you call a fruitful function all by itself, the return value is lost forever!

```
math.sqrt(5)
```

This script computes the square root of 5, but since it doesn't store or display the result, it is not very useful.

Void functions might display something on the screen or have some other effect, but they don't have a return value. If you try to assign the result to a variable, you get a special value called `None`.

```
>>> result = print_twice('Bing')
Bing
Bing
>>> print(result)
None
```

The value `None` is not the same as the string `'None'`. It is a special value that has its own class:

```
>>> print(type(None))
<class 'NoneType'>
```

The functions we have written so far are all void. We will start writing fruitful functions in the next section.

5.9 Return values

Our first example of a fruitful function is `area`, which returns the area of a circle with the given radius:

```
def area(radius):
    temp = math.pi * radius**2
    return temp
```

In a fruitful function the `return` statement includes a return value. This statement means: “Return immediately from this function and use the following expression as a return value.” The expression provided can be arbitrarily complicated, so we could have written this function more concisely:

```
def area(radius):
    return math.pi * radius**2
```

On the other hand, **temporary variables** like `temp` often make debugging easier.

Sometimes it is useful to have multiple return statements, one in each branch of a conditional:

```
def absolute_value(x):
    if x < 0:
        return -x
    else:
        return x
```

Since these `return` statements are in an alternative conditional, only one will be executed.

As soon as a return statement executes, the function terminates without executing any subsequent statements. Code that appears after a `return` statement, or any other place the flow of execution can never reach, is called **dead code**.

In a fruitful function, it is a good idea to ensure that every possible path through the program hits a `return` statement. For example:

```
def absolute_value(x):
    if x < 0:
        return -x
    elif x > 0:
        return x
```

This program is not correct because if `x` happens to be 0, neither condition is true, and the function ends without hitting a `return` statement. If the flow of execution gets to the end of a function, the return value is `None`, which is not the absolute value of 0.

```
>>> print(absolute_value(0))
None
```

Exercise 5.3. *Write a* `compare` *function that returns* `1` *if* `x > y`, `0` *if* `x == y`, *and* `-1` *if* `x < y`.

5.10 Boolean functions

Functions can return booleans, which is often convenient for hiding complicated tests inside functions. For example:

```
def is_divisible(x, y):
    if x % y == 0:
        return True
    else:
        return False
```

It is common to give boolean functions names that sound like yes/no questions; `is_divisible` returns either `True` or `False` to indicate whether `x` is divisible by `y`.

Here is an example:

```
>>>   is_divisible(6, 4)
False
>>>   is_divisible(6, 3)
True
```

The result of the == operator is a boolean, so we can write the function more concisely by returning it directly:

```
def is_divisible(x, y):
    return x % y == 0
```

Boolean functions are often used in conditional statements:

```
if is_divisible(x, y):
    print('x is divisible by y')
```

It might be tempting to write something like:

```
if is_divisible(x, y) == True:
    print('x is divisible by y')
```

But the extra comparison is unnecessary.
Exercise 5.4. *Write a function* `is_between(x, y, z)` *that returns* `True` *if $x \leq y \leq z$ or* `False` *otherwise.*

5.11 Incremental development

As you write larger functions, you might start find yourself spending more time debugging.

To deal with increasingly complex programs, you might want to try a process called **incremental development**. The goal of incremental development is to avoid long debugging sessions by adding and testing only a small amount of code at a time.

As an example, suppose you want to find the distance between two points, given by the coordinates (x_1, y_1) and (x_2, y_2). By the Pythagorean theorem, the distance is:

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The first step is to consider what a `distance` function should look like in Python. In other words, what are the inputs (parameters) and what is the output (return value)?

In this case, the two points are the inputs, which you can represent using four parameters. The return value is the distance, which is a floating-point value.

Already you can write an outline of the function:

```
def distance(x1, y1, x2, y2):
    return 0.0
```

Obviously, this version doesn't compute distances; it always returns zero. But it is syntactically correct, and it runs, which means that you can test it before you make it more complicated.

To test the new function, call it with sample arguments:

```
>>> distance(1, 2, 4, 6)
0.0
```

I chose these values so that the horizontal distance is 3 and the vertical distance is 4; that way, the result is 5 (the hypotenuse of a 3-4-5 triangle). When testing a function, it is useful to know the right answer.

At this point we have confirmed that the function is syntactically correct, and we can start adding code to the body. A reasonable next step is to find the differences $x_2 - x_1$ and $y_2 - y_1$. The next version stores those values in temporary variables and prints them.

```
def distance(x1, y1, x2, y2):
    dx = x2 - x1
    dy = y2 - y1
    print('dx is', dx)
    print('dy is', dy)
    return 0.0
```

If the function is working, it should display `'dx is 3'` and `'dy is 4'`. If so, we know that the function is getting the right arguments and performing the first computation correctly. If not, there are only a few lines to check.

Next we compute the sum of squares of `dx` and `dy`:

```
def distance(x1, y1, x2, y2):
    dx = x2 - x1
    dy = y2 - y1
    dsquared = dx**2 + dy**2
    print('dsquared is: ', dsquared)
    return 0.0
```

Again, you would run the program at this stage and check the output (which should be 25).

Finally, you can use `math.sqrt` to compute and return the result:

```
def distance(x1, y1, x2, y2):
    dx = x2 - x1
    dy = y2 - y1
    dsquared = dx**2 + dy**2
    result = math.sqrt(dsquared)
    return result
```

If that works correctly, you are done. Otherwise, you might want to print the value of `result` before the return statement.

The final version of the function doesn't display anything when it runs; it only returns a value. The `print` statements we wrote are useful for debugging, but once you get the function working, you should remove them. Code like that is called **scaffolding** because it is helpful for building the program but is not part of the final product.

When you start out, you should add only a line or two of code at a time. As you gain more experience, you might find yourself writing and debugging bigger chunks. Either way, incremental development can save you a lot of debugging time.

The key aspects of the process are:

1. Start with a working program and make small incremental changes. At any point, if there is an error, you should have a good idea where it is.

2. Use temporary variables to hold intermediate values so you can display and check them.

3. Once the program is working, you might want to remove some of the scaffolding or consolidate multiple statements into compound expressions, but only if it does not make the program difficult to read.

Exercise 5.5. *Use incremental development to write a function called* `hypotenuse` *that returns the length of the hypotenuse of a right triangle given the lengths of the two legs as arguments. Record each stage of the development process as you go.*

5.12 docstring

A **docstring** is a string at the beginning of a function that explains the interface ("doc" is short for "documentation"). Here is an example for our above distance function:

```
def distance(x1, y1, x2, y2):
    """Calculates the distance between two points
    when given their x and y numeric values.
    """
    dx = x2 - x1
    dy = y2 - y1
    dsquared = dx**2 + dy**2
    result = math.sqrt(dsquared)
    return result
```

This docstring is a triple-quoted string, also known as a multi-line string because the triple quotes allow the string to span more than one line.

It is terse, but it contains the essential information someone would need to use this function. It explains concisely what the function does (without getting into the details of how it does it). It explains what effect each parameter has on the behavior of the function and what type each parameter should be (if it is not obvious).

Writing this kind of documentation is an important part of interface design. A well-designed interface should be simple to explain; if you are having a hard time explaining one of your functions, that might mean that the interface could be improved.

5.13 Composition

As you should expect by now, you can call one function from within another. This ability is called **composition**.

As an example, we'll write a function that takes two points, the center of the circle and a point on the perimeter, and computes the area of the circle.

Assume that the center point is stored in the variables `xc` and `yc`, and the perimeter point is in `xp` and `yp`. The first step is to find the radius of the circle, which is the distance between the two points. Fortunately, there is a function, `distance`, that does that:

```
radius = distance(xc, yc, xp, yp)
```

The next step is to find the area of a circle with that radius:

```
result = area(radius)
```

Wrapping that up in a function, we get:

```
def circle_area(xc, yc, xp, yp):
    radius = distance(xc, yc, xp, yp)
    result = area(radius)
    return result
```

The temporary variables `radius` and `result` are useful for development and debugging, but once the program is working, we can make it more concise by composing the function calls:

```
def circle_area(xc, yc, xp, yp):
    return area(distance(xc, yc, xp, yp))
```

5.14 Debugging

As you start writing bigger programs, you might find yourself spending more time debugging. More code means more chances to make an error and more place for bugs to hide.

One way to cut your debugging time is “debugging by bisection.” For example, if there are 100 lines in your program and you check them one at a time, it would take 100 steps.

Instead, try to break the problem in half. Look at the middle of the program, or near it, for an intermediate value you can check. Add a `print` statement (or something else that has a verifiable effect) and run the program.

If the mid-point check is incorrect, the problem must be in the first half of the program. If it is correct, the problem is in the second half.

Every time you perform a check like this, you halve the number of lines you have to search. After six steps (which is much less than 100), you would be down to one or two lines of code.

At least in theory. In practice it is not always clear what the “middle of the program” is and not always possible to check it. It doesn’t make sense to count lines and find the exact midpoint. Instead, think about places in the program where there might be errors and places where it is easy to put a check. Then choose a spot where you think the chances are about the same that the bug is before or after the check.

5.15 Glossary

fruitful function: A function that returns a value.

void function: A function that doesn’t return a value.

function definition: A statement that creates a new function, specifying its name, parameters, and the statements it executes.

function object: A value created by a function definition. The name of the function is a variable that refers to a function object.

header: The first line of a function definition.

body: The sequence of statements inside a function definition.

parameter: A name used inside a function to refer to the value passed as an argument.

function call: A statement that executes a function. It consists of the function name followed by an argument list.

argument: A value provided to a function when the function is called. This value is assigned to the corresponding parameter in the function.

local variable: A variable defined inside a function. A local variable can only be used inside its function.

return value: The result of a function. If a function call is used as an expression, the return value is the value of the expression.

flow of execution: The order in which statements are executed during a program run.

stack diagram: A graphical representation of a stack of functions, their variables, and the values they refer to.

frame: A box in a stack diagram that represents a function call. It contains the local variables and parameters of the function.

traceback: A list of the functions that are executing, printed when an exception occurs.

temporary variable: A variable used to store an intermediate value in a complex calculation.

dead code: Part of a program that can never be executed, often because it appears after a `return` statement.

`None`**:** A special value returned by functions that have no return statement or a return statement without an argument.

incremental development: A program development plan intended to avoid debugging by adding and testing only a small amount of code at a time.

scaffolding: Code that is used during program development but is not part of the final version.

guardian: A programming pattern that uses a conditional statement to check for and handle circumstances that might cause an error.

5.16 Exercises

Exercise 5.6. *Fermat's Last Theorem says that there are no integers a, b, and c such that*

$$a^n + b^n = c^n$$

for any values of n greater than 2.

Write a function named `check_fermat` *that takes four parameters—*`a`*,* `b`*,* `c` *and* `n`*—and that checks to see if Fermat's theorem holds. If n is greater than 2 and it turns out to be true that*

$$a^n + b^n = c^n$$

the program should print "Holy smokes, Fermat was wrong!" Otherwise the program should print "No, that doesn't work."

Exercise 5.7. *Python provides a built-in function called* `len` *that returns the length of a string, so the value of* `len('allen')` *is 5.*

Write a function named `right_justify` *that takes a string named* `s` *as a parameter and that prints the string with enough leading spaces so that the last letter of the string is in column 70 of the display.*

```
>>> right_justify('allen')
                                                                 allen
```

Exercise 5.8.

Write a function that draws grids like this in any size[1]*:*

```
+ - - - - - + - - - - - +
|           |           |
|           |           |
|           |           |
|           |           |
+ - - - - - + - - - - - +
|           |           |
|           |           |
|           |           |
|           |           |
+ - - - - - + - - - - - +
```

Hint: to print more than one value on a line, you can print a comma-separated sequence:

```
print('+', '-')
```

[1]Based on an exercise in Oualline, *Practical C Programming, Third Edition*, O'Reilly (1997)

If we add the argument `end=''`*, Python leaves the line unfinished, so the value printed next appears on the same line.*

```
print('+', end='')
print('-')
```

The output of these statements is `'+ -'`.

Chapter 6

Strings

6.1 Characters

A character is a string one unit in length. Characters are stored internally in the computer as a number, with one unique number for each character. We can find this number with the `ord` conversion function, and conversely convert numbers into characters with the `chr` function.

```
>>> ord('A')
65
>>> chr(66)
'B'
```

6.2 A string is a sequence

A string is a **sequence** of characters. You can access the characters one at a time with the bracket operator:

```
>>> fruit = 'banana'
>>> letter = fruit[1]
```

The second statement selects character number 1 from `fruit` and assigns it to `letter`.

The expression in brackets is called an **index**. The index *indicates* which character in the sequence you want (hence the name).

But you might not get what you expect:

```
>>> print(letter)
a
```

For most people, the first letter of `'banana'` is `b`, not `a`. But for computer scientists, the index is an offset from the beginning of the string, and the offset of the first letter is zero.

```
>>> letter = fruit[0]
>>> print(letter)
b
```

So `b` is the 0th letter ("zero-eth") of `'banana'`, `a` is the 1th letter ("one-eth"), and `n` is the 2th ("two-eth") letter.

You can use any expression, including variables and operators, as an index, but the value of the index has to be an integer. Otherwise you get:

```
>>> letter = fruit[1.0]
TypeError: string indices must be integers
```

6.3 `len`

`len` is a built-in function that returns the number of characters in a string:

```
>>> fruit = 'banana'
>>> len(fruit)
6
```

To get the last letter of a string, you might be tempted to try something like this:

```
>>> length = len(fruit)
>>> last = fruit[length]
IndexError: string index out of range
```

The reason for the `IndexError` is that there is no letter in `'banana'` with the index 6. Since we started counting at zero, the six letters are numbered 0 to 5. To get the last character, you have to subtract 1 from `length`:

```
>>> last = fruit[length-1]
>>> print(last)
a
```

Alternatively, you can use negative indices, which count backward from the end of the string. The expression `fruit[-1]` yields the last letter, `fruit[-2]` yields the second to last, and so on.

6.4 String operations

In general, you cannot perform mathematical operations on strings, even if the strings look like numbers, so the following are illegal:

```
'2'-'1'    'eggs'/'easy'    'third'*'a charm'
```

The + operator does work with strings, but it might not do exactly what you expect: it performs **concatenation**, which means joining the strings by linking them end-to-end. For example:

```
first = 'throat'
second = 'warbler'
print(first + second)
```

The output of this program is `throatwarbler`.

The `*` operator also works on strings; it performs repetition. For example, `'Spam'*3` is `'SpamSpamSpam'`. If one of the operands is a string, the other has to be an integer.

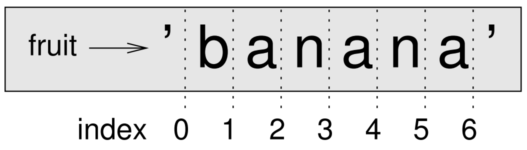
On one hand, this use of `+` and `*` makes sense by analogy with addition and multiplication. Just as `4*3` is equivalent to `4+4+4`, we expect `'Spam'*3` to be the same as `'Spam'+'Spam'+'Spam'`, and it is. On the other hand, there is a significant way in which string concatenation and repetition are different from integer addition and multiplication. Can you think of a property that addition and multiplication have that string concatenation and repetition do not?

6.5 String slices

A segment of a string is called a **slice**. Selecting a slice is similar to selecting a character:

```
>>> s = 'Monty Python'
>>> print(s[0:5])
Monty
>>> print(s[6:13])
Python
```

The operator `[n:m]` returns the part of the string from the “n-eth” character to the “m-eth” character, including the first but excluding the last. This behavior is counterintuitive, but might help to imagine the indices pointing *between* the characters, as in the following diagram:



If you omit the first index (before the colon), the slice starts at the beginning of the string. If you omit the second index, the slice goes to the end of the string. Thus:

```
>>> fruit = 'banana'
>>> fruit[:3]
'ban'
>>> fruit[3:]
'ana'
```

If the first index is greater than or equal to the second the result is an **empty string**, represented by two quotation marks:

```
>>> fruit = 'banana'
>>> fruit[3:3]
''
```

An empty string contains no characters and has length 0, but other than that, it is the same as any other string.

Exercise 6.1. *Given that* `fruit` *is a string, what does* `fruit[:]` *mean?*

6.6 Strings are immutable

It is tempting to use the `[]` operator on the left side of an assignment, with the intention of changing a character in a string. For example:

```
>>> greeting = 'Hello, world!'
>>> greeting[0] = 'J'
TypeError: object does not support item assignment
```

The "object" in this case is the string and the "item" is the character you tried to assign. For now, an **object** is the same thing as a value, but we will refine that definition later. An **item** is one of the values in a sequence.

The reason for the error is that strings are **immutable**, which means you can't change an existing string. The best you can do is create a new string that is a variation on the original:

```
>>> greeting = 'Hello, world!'
>>> new_greeting = 'J' + greeting[1:]
>>> print(new_greeting)
Jello, world!
```

This example concatenates a new first letter onto a slice of `greeting`. It has no effect on the original string.

6.7 `string` methods

A **method** is similar to a function—it takes arguments and returns a value—but the syntax is different. Methods are attached to classes. For example, the method `upper` is a part of the string class and returns a new string with all uppercase letters:

Instead of the function syntax `upper(word)`, it uses the method syntax `word.upper()`.

```
>>> word = 'banana'
>>> new_word = word.upper()
>>> print(new_word)
BANANA
```

This form of dot notation specifies the name of the method, `upper`, and the name of the string to apply the method to, `word`. The parentheses indicate that this method has no parameters.

A method call is called an **invocation**; in this case, we would say that we are invoking `upper` on the `word`.

The string method named `find` is the opposite of the `[]` operator. Instead of taking an index and extracting the corresponding character, it takes a character and finds the index where that character appears. If the character is not found, the function returns `-1`.

```
>>> word = 'banana'
>>> index = word.find('a')
>>> print(index)
1
```

In this example, we invoke `find` on `word` and pass the letter we are looking for as a parameter.

The `find` method can find substrings, not just characters:

```
>>> word.find('na')
2
```

It can take as a second argument the index where it should start:

```
>>> word.find('na', 3)
4
```

And as a third argument where it should stop:

```
>>> name = 'bob'
>>> name.find('b', 1, 2)
-1
```

This search fails because `b` does not appear in the index range from `1` to `2` (not including `2`).
Exercise 6.2. *Another useful string method is called* `count` *Read the documentation of this method and write an invocation that counts the number of* `a`s *in* `'banana'`. *Hint: there are three.*

6.8 Glossary

object: Something a variable can refer to. For now, you can use “object” and “value” interchangeably.

sequence: An ordered set; that is, a set of values where each value is identified by an integer index.

item: One of the values in a sequence.

index: An integer value used to select an item in a sequence, such as a character in a string.

slice: A part of a string specified by a range of indices.

empty string: A string with no characters and length 0, represented by two quotation marks.

concatenate: To join two operands end-to-end.

immutable: The property of a sequence whose items cannot be assigned.

traverse: To iterate through the items in a sequence, performing a similar operation on each.

search: A pattern of traversal that stops when it finds what it is looking for.

counter: A variable used to count something, usually initialized to zero and then incremented.

method: A function that is associated with an object and called using dot notation.

invocation: A statement that calls a method.

6.9 Exercises

Chapter 7

Iteration

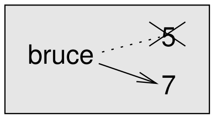
7.1 Multiple assignment

As you may have discovered, it is legal to make more than one assignment to the same variable. A new assignment makes an existing variable refer to a new value (and stop referring to the old value).

```
bruce = 5
print(bruce)
bruce = 7
print(bruce)
```

The output of this program is `5`, then `7`, because the first time `bruce` is printed, its value is 5, and the second time, its value is 7. The comma at the end of the first `print` statement suppresses the newline, which is why both outputs appear on the same line.

Here is what **multiple assignment** looks like in a state diagram:



With multiple assignment it is especially important to distinguish between an assignment operation and a statement of equality. Because Python uses the equal sign (=) for assignment, it is tempting to interpret a statement like `a = b` as a statement of equality. It is not!

First, equality is a symmetric relation and assignment is not. For example, in mathematics, if $a = 7$ then $7 = a$. But in Python, the statement `a = 7` is legal and `7 = a` is not.

Furthermore, in mathematics, a statement of equality is either true or false, for all time. If $a = b$ now, then a will always equal b. In Python, an assignment statement can make two variables equal, but they don't have to stay that way:

```
a = 5
b = a    # a and b are now equal
a = 3    # a and b are no longer equal
```

The third line changes the value of `a` but does not change the value of `b`, so they are no longer equal.

Although multiple assignment is frequently helpful, you should use it with caution. If the values of variables change frequently, it can make the code difficult to read and debug.

7.2 Updating variables

One of the most common forms of multiple assignment is an **update**, where the new value of the variable depends on the old.

```
x = x+1
```

This means "get the current value of `x`, add one, and then update `x` with the new value."

If you try to update a variable that doesn't exist, you get an error, because Python evaluates the right side before it assigns a value to `x`:

```
>>> x = x + 1
NameError: name 'x' is not defined
```

Before you can update a variable, you have to **initialize** it, usually with a simple assignment:

```
>>> x = 0
>>> x = x + 1
```

Updating a variable by adding 1 is called an **increment**; subtracting 1 is called a **decrement**.

Since updating variables is so common, there is a syntax shortcut for these operations. We can rewrite `x = x + 1` as `x +=1`, where the operator immediately precedes the assignment.

```
>>> x = 0
>>> x += 1
```

This shortcut works will all of our typical operators, `+`, `-`, `*`, `/`, `//` and `%`.

7.3 The `while` statement

Computers are often used to automate repetitive tasks. Repeating identical or similar tasks without making errors is something that computers do well and people do poorly.

This repetition is also called **iteration**. Because iteration is so common, Python provides several language features to make it easier. One is the `while` statement. Here is a function called `countdown` that uses a `while` statement to simulate a rocket launch countdown:

```
def countdown(n):
    while n > 0:
        print(n)
        n = n - 1
    print('Blastoff!')
```

You can almost read the `while` statement as if it were English. It means, “While `n` is greater than 0, display the value of `n` and then reduce the value of `n` by 1. When you get to 0, display the word `Blastoff!`”

More formally, here is the flow of execution for a `while` statement:

1. Evaluate the condition, yielding `True` or `False`.
2. If the condition is false, exit the `while` statement and continue execution at the next statement.
3. If the condition is true, execute the body and then go back to step 1.

This type of flow is called a **loop** because the third step loops back around to the top.

The body of the loop should change the value of one or more variables so that eventually the condition becomes false and the loop terminates. Otherwise the loop will repeat forever, which is called an **infinite loop**. An endless source of amusement for computer scientists is the observation that the directions on shampoo, “Lather, rinse, repeat,” are an infinite loop.

In the case of `countdown`, we can prove that the loop terminates because we know that the value of `n` is finite, and we can see that the value of `n` gets smaller each time through the loop, so eventually we have to get to 0. In other cases, it is not so easy to tell:

```
def sequence(n):
    while n != 1:
        print(n)
        if n % 2 == 0:        # n is even
            n = n / 2
        else:                # n is odd
            n = n * 3 + 1
```

The condition for this loop is `n != 1`, so the loop will continue until `n` is `1`, which makes the condition false.

Each time through the loop, the program outputs the value of `n` and then checks whether it is even or odd. If it is even, `n` is divided by 2. If it is odd, the value of `n` is replaced with `n*3+1`. For example, if the argument passed to `sequence` is 3, the resulting sequence is 3, 10, 5, 16, 8, 4, 2, 1.

Since `n` sometimes increases and sometimes decreases, there is no obvious proof that `n` will ever reach 1, or that the program terminates. For some particular values of `n`, we can prove termination. For example, if the starting value is a power of two, then the value of `n` will be even each time through the loop until it reaches 1. The previous example ends with such a sequence, starting with 16.

The hard question is whether we can prove that this program terminates for *all positive values* of `n`. So far, no one has been able to prove it *or* disprove it!

7.4 Sentinel loops

Sometimes you don't know it's time to end a loop until you get half way through the body. In that case you can set a **sentinel** to watch for a condition and jump out of the loop.

For example, suppose you want to take input from the user until they type `done`. You could write:

```
finished = False
while not finished:
    line = input('> ')
    if line == 'done':
        finished = True
    else:
        print(line)

print('Done!')
```

The loop condition is based on the sentinel `finished`, which begins as `False`, meaning we are not finished with the loop.

Each time through, it prompts the user with an angle bracket. If the user types `done`, the sentinel activates and will be set to `True`, which exits the loop. Otherwise the program echos whatever the user types and goes back to the top of the loop. Here's a sample run:

```
> not done
not done
> done
Done!
```

This way of writing `while` loops is common because you can check the condition in multiple ways anywhere in the loop (not just at the top).

7.5 Square roots

Loops are often used in programs that compute numerical results by starting with an approximate answer and iteratively improving it.

For example, one way of computing square roots is Newton's method. Suppose that you want to know the square root of a. If you start with almost any estimate, x, you can compute a better estimate with the following formula:

$$y = \frac{x + a/x}{2}$$

For example, if a is 4 and x is 3:

```
>>> a = 4.0
>>> x = 3.0
>>> y = (x + a / x) / 2
>>> print(y)
2.16666666667
```

Which is closer to the correct answer ($\sqrt{4} = 2$). If we repeat the process with the new estimate, it gets even closer:

```
>>> x = y
>>> y = (x + a / x) / 2
>>> print(y)
2.00641025641
```

After a few more updates, the estimate is almost exact:

```
>>> x = y
>>> y = (x + a / x) / 2
>>> print(y)
2.00001024003
>>> x = y
>>> x = (x + a / x) / 2
>>> print(y)
2.00000000003
```

In general we don't know ahead of time how many steps it takes to get to the right answer, but we know when we get there because the estimate stops changing:

```
>>> x = y
>>> y = (x + a / x) / 2
>>> print(y)
2.0
>>> x = y
>>> y = (x + a / x) / 2
>>> print(y)
2.0
```

When `y == x`, we can stop. Here is a loop that starts with an initial estimate, `x`, and improves it until it stops changing:

```
finished = False
while not finished:
    print(x)
    y = (x + a / x) / 2
    if y == x:
        finished = True
    x = y
```

For most values of `a` this works fine, but in general it is dangerous to test `float` equality. Floating-point values are only approximately right: most rational numbers, like $1/3$, and irrational numbers, like $\sqrt{2}$, can't be represented exactly with a `float`.

Rather than checking whether `x` and `y` are exactly equal, it is safer to use `math.fabs` to compute the absolute value, or magnitude, of the difference between them:

```
    if math.fabs(y - x) < something_small:
        finished = True
```

Where `something_small` has a value like `0.0000001` that determines how close is close enough.
Exercise 7.1. *Wrap this loop in a function called* `square_root` *that takes* `a` *as a parameter, chooses a reasonable value of* `x`*, and returns an estimate of the square root of* `a`.

7.6 Debugging

When you use indices to traverse the values in a sequence, it is tricky to get the beginning and end of the traversal right. Here is a function that is supposed to compare two words and return `True` if one of the words is the reverse of the other, but it contains two errors:

```
def is_reverse(word1, word2):
    if len(word1) != len(word2):
        return False

    i = 0
    j = len(word2)

    while j > 0:
        if word1[i] != word2[j]:
            return False
        i = i + 1
        j = j - 1

    return True
```

The first `if` statement checks whether the words are the same length. If not, we can return `False` immediately and then, for the rest of the function, we can assume that the words are the same length. This is another example of a guardian.

`i` and `j` are indices: `i` traverses `word1` forward while `j` traverses `word2` backward. If we find two letters that don't match, we can return `False` immediately. If we get through the whole loop and all the letters match, we return `True`.

If we test this function with the words "pots" and "stop", we expect the return value `True`, but we get an IndexError:

```
>>> is_reverse('pots', 'stop')
...
  File "reverse.py", line 15, in is_reverse
    if word1[i] != word2[j]:
IndexError: string index out of range
```

For debugging this kind of error, my first move is to print the values of the indices immediately before the line where the error appears.

```
    while j > 0:
        print(i, j)        # print here

        if word1[i] != word2[j]:
            return False
        i = i + 1
        j = j - 1
```

Now when I run the program again, I get more information:

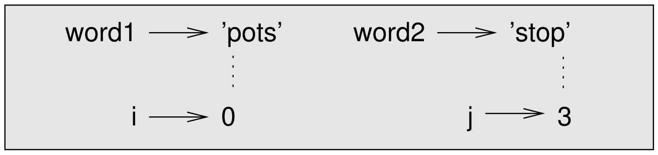
```
>>> is_reverse('pots', 'stop')
0 4
...
IndexError: string index out of range
```

The first time through the loop, the value of `j` is 4, which is out of range for the string `'pots'`. The index of the last character is 3, so the initial value for `j` should be `len(word2) - 1`.

If I fix that error and run the program again, I get:

```
>>> is_reverse('pots', 'stop')
0 3
1 2
2 1
True
```

This time we get the right answer, but it looks like the loop only ran three times, which is suspicious. To get a better idea of what is happening, it is useful to draw a state diagram. During the first iteration, the frame for `is_reverse` looks like this:



I took a little license by arranging the variables in the frame and adding dotted lines to show that the values of `i` and `j` indicate characters in `word1` and `word2`.

7.7 Glossary

multiple assignment: Making more than one assignment to the same variable during the execution of a program.

update: An assignment where the new value of the variable depends on the old.

initialize: An assignment that gives an initial value to a variable that will be updated.

increment: An update that increases the value of a variable (often by one).

decrement: An update that decreases the value of a variable.

iteration: Repeated execution of a set of statements using a loop.

infinite loop: A loop in which the terminating condition is never satisfied.

7.8 Exercises

Exercise 7.2. *To test the square root algorithm in this chapter, you could compare it with* `math.sqrt`*. Write a function named* `test_square_root` *that prints a table like this:*

```
1.0 1.0           1.0           0.0
2.0 1.41421356237 1.41421356237 2.22044604925e-16
3.0 1.73205080757 1.73205080757 0.0
4.0 2.0           2.0           0.0
5.0 2.2360679775  2.2360679775  0.0
6.0 2.44948974278 2.44948974278 0.0
7.0 2.64575131106 2.64575131106 0.0
8.0 2.82842712475 2.82842712475 4.4408920985e-16
9.0 3.0           3.0           0.0
```

The first column is a number, a; the second column is the square root of a computed with the function from Exercise 7.1; the third column is the square root computed by `math.sqrt`*; the fourth column is the absolute value of the difference between the two estimates.*

Exercise 7.3. *The built-in function* `eval` *takes a string and evaluates it using the Python interpreter. For example:*

```
>>> eval('1 + 2 * 3')
7
>>> import math
>>> eval('math.sqrt(5)')
2.2360679774997898
>>> eval('type(math.pi)')
<class 'float'>
```

Write a function called `eval_loop` *that iteratively prompts the user, takes the resulting input and evaluates it using* `eval`*, and prints the result.*

It should continue until the user enters `'done'`*, and then return the value of the last expression it evaluated.*

Chapter 8

Lists

8.1 A list is a sequence

Like a string, a **list** is a sequence of values. In a string, the values are characters; in a list, they can be any type. The values in list are called **elements** or sometimes **items**.

There are several ways to create a new list; the simplest is to enclose the elements in square brackets (`[` and `]`):

```
[10, 20, 30, 40]
['crunchy frog', 'ram bladder', 'lark vomit']
```

The first example is a list of four integers. The second is a list of three strings. The elements of a list don't have to be the same type. The following list contains a string, a float, an integer, and (lo!) another list:

```
['spam', 2.0, 5, [10, 20]]
```

A list within another list is said to be **nested**.

A list that contains no elements is called an empty list; you can create one with empty brackets, `[]`.

As you might expect, you can assign list values to variables:

```
>>> cheeses = ['Cheddar', 'Edam', 'Gouda']
>>> numbers = [17, 123]
>>> empty = []
>>> print(cheeses, numbers, empty)
['Cheddar', 'Edam', 'Gouda'] [17, 123] []
```

8.2 Lists are mutable

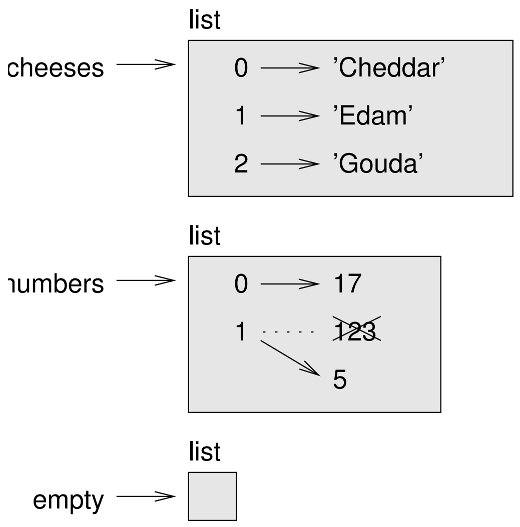
The syntax for accessing the elements of a list is the same as for accessing the characters of a string—the bracket operator ([]). The expression inside the brackets specifies the index. Remember that the indices start at 0:

```
>>> print(cheeses[0])
Cheddar
```

Unlike strings, lists are mutable. When the bracket operator appears on the left side of an assignment, it identifies the element of the list that will be assigned.

```
>>> numbers = [17, 123]
>>> numbers[1] = 5
>>> print(numbers)
[17, 5]
```

You can think of a list as a relationship between indices and elements. This relationship is called a **mapping**; each index "maps to" one of the elements. Here is a state diagram showing `cheeses`, `numbers` and `empty`:



Lists are represented by boxes with the word "list" outside and the elements of the list inside. `cheeses` refers to a list with three elements indexed 0, 1 and 2. `numbers` contains two elements; the diagram shows that the value of the second element has been reassigned from 123 to 5. `empty` refers to a list with no elements.

The bracket operator can appear anywhere in an expression. When it appears on the left side of an assignment, it changes one of the elements in the list, so the one-eth element of `numbers`, which used to be 123, is now 5.

List indices work the same way as string indices:

- Any integer expression can be used as an index.
- If you try to read or write an element that does not exist, you get an `IndexError`.
- If an index has a negative value, it counts backward from the end of the list.

8.3 List operations

The + operator concatenates lists:

```
>>> a = [1, 2, 3]
>>> b = [4, 5, 6]
>>> c = a + b
>>> print(c)
[1, 2, 3, 4, 5, 6]
```

Similarly, the `*` operator repeats a list a given number of times:

```
>>> [0] * 4
[0, 0, 0, 0]
>>> [1, 2, 3] * 3
[1, 2, 3, 1, 2, 3, 1, 2, 3]
```

The first example repeats `[0]` four times. The second example repeats the list `[1, 2, 3]` three times.

8.4 List slices

The slice operator also work on lists:

```
>>> t = ['a', 'b', 'c', 'd', 'e', 'f']
>>> t[1:3]
['b', 'c']
>>> t[:4]
['a', 'b', 'c', 'd']
>>> t[3:]
['d', 'e', 'f']
```

If you omit the first index, the slice starts at the beginning. If you omit the second, the slice goes to the end. So if you omit both, the slice is a copy of the whole list.

```
>>> t[:]
['a', 'b', 'c', 'd', 'e', 'f']
```

A slice operator on the left side of an assignment can update multiple elements:

```
>>> t = ['a', 'b', 'c', 'd', 'e', 'f']
>>> t[1:3] = ['x', 'y']
>>> print(t)
['a', 'x', 'y', 'd', 'e', 'f']
```

8.5 List methods

Python provides methods that operate on lists. For example, `append` adds a new element to the end of a list:

```
>>> t = ['a', 'b', 'c']
>>> t.append('d')
>>> print(t)
['a', 'b', 'c', 'd']
```

`extend` takes a list as an argument and appends all of the elements:

```
>>> t1 = ['a', 'b', 'c']
>>> t2 = ['d', 'e']
>>> t1.extend(t2)
>>> print(t1)
['a', 'b', 'c', 'd', 'e']
```

This example leaves `t2` unmodified.

`sort` arranges the elements of the list from low to high:

```
>>> t = ['d', 'c', 'e', 'b', 'a']
>>> t.sort()
>>> print(t)
['a', 'b', 'c', 'd', 'e']
```

List methods are all void; they modify the list and return `None`. If you accidentally write `t = t.sort()`, you will be disappointed with the result.

8.6 Deleting elements

There are several ways to delete elements from a list. If you know the index of the element you want, you can use `pop`:

```
>>> t = ['a', 'b', 'c']
>>> x = t.pop(1)
>>> print(t)
['a', 'c']
>>> print(x)
b
```

`pop` modifies the list and returns the element that was removed.

If you don't need the removed value, you can use the `del` operator:

```
>>> t = ['a', 'b', 'c']
>>> del t[1]
>>> print(t)
['a', 'c']
```

If you know the element you want to remove (but not the index), you can use `remove`:

```
>>> t = ['a', 'b', 'c']
>>> t.remove('b')
>>> print(t)
['a', 'c']
```

The return value from `remove` is `None`.

To remove more than one element, you can use `del` with a slice index:

```
>>> t = ['a', 'b', 'c', 'd', 'e', 'f']
>>> del t[1:5]
>>> print(t)
['a', 'f']
```

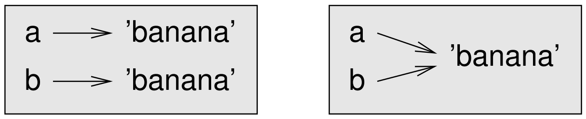
As usual, the slice selects all the elements up to, but not including, the second index.

8.7 Objects and values

If we execute these assignment statements:

```
a = 'banana'
b = 'banana'
```

We know that `a` and `b` both refer to a string, but we don't know whether they refer to the *same* string. There are two possible states:



In one case, `a` and `b` refer to two different objects that have the same value. In the second case, they refer to the same object.

To check whether two variables refer to the same object, you can use the `is` operator.

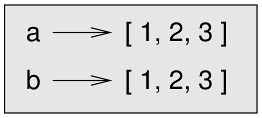
```
>>> a = 'banana'
>>> b = 'banana'
>>> a is b
True
```

In this example, Python only created one string object, and both `a` and `b` refer to it.

In contrast, when you create two lists, you get two objects:

```
>>> a = [1, 2, 3]
>>> b = [1, 2, 3]
>>> a is b
False
```

So the state diagram looks like this:



In this case we would say that the two lists are **equivalent**, because they have the same elements, but not **identical**, because they are not the same object. If two objects are identical, they are also equivalent, but if they are equivalent, they are not necessarily identical.

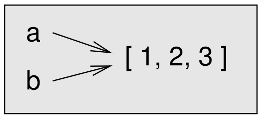
Until now, we have been using "object" and "value" interchangeably, but it is more precise to say that an object has a value. If you execute `a = [1,2,3]`, `a` refers to a list object whose value is a particular sequence of elements. If another list has the same elements, we would say it has the same value.

8.8 Aliasing

If `a` refers to an object and you assign `b = a`, then both variables refer to the same object. For example, if you execute:

```
>>> a = [1, 2, 3]
>>> b = a
```

Then `a` and `b` refer to the same list. The state diagram looks like this:



The association of a variable with an object is called a **reference**. In this example, there are two references to the same object.

An object with more than one reference has, in some sense, more than one name, so we say that the object is **aliased**.

If the aliased object is mutable, changes made with one alias affect the other:

```
>>> b[0] = 17
>>> print(a)
[17, 2, 3]
```

Although this behavior can be useful, it is sometimes unexpected or undesirable. In general, it is safer to avoid aliasing when you are working with mutable objects.

For immutable objects like strings, aliasing is not as much of a problem. In this example:

```
a = 'banana'
b = 'banana'
```

It almost never makes a difference whether `a` and `b` refer to the same string or not.

8.9 List arguments

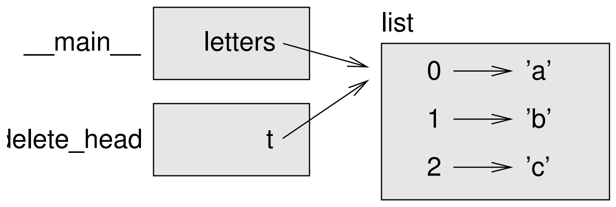
When you pass a list to a function, the function gets a reference to the list. If the function modifies a list parameter, the caller sees the change. For example, `delete_head` removes the first element from a list:

```
def delete_head(t):
    del t[0]
```

Here's how it is used:

```
>>> letters = ['a', 'b', 'c']
>>> delete_head(letters)
>>> print(letters)
['b', 'c']
```

The parameter `t` and the variable `letters` are aliases for the same object. The stack diagram looks like this:



Since the list is shared by two frames, I drew it between them.

If a function returns a list, it returns a reference to the list. For example, `tail` returns a list that contains all but the first element of the given list:

```
def tail(t):
    return t[1:]
```

Here's how `tail` is used:

```
>>> letters = ['a', 'b', 'c']
>>> rest = tail(letters)
>>> print(rest)
['b', 'c']
```

Because the return value was created with the slice operator, it is a new list. The original list is unmodified.

8.10 Copying lists

When you assign an object to a variable, Python copies the reference to the object.

```
>>> a = [1, 2, 3]
>>> b = a
```

In this case `a` and `b` refer to the same list.

If you want to copy the list (not just a reference to it), you can use the slice operator:

```
>>> a = [1, 2, 3]
>>> b = a[:]
>>> print(b)
[1, 2, 3]
```

Making a slice of `a` creates a new list. In this case the slice contains all of the elements from the original list.

Another way to make a copy is the `copy` function from the `copy` module:

```
>>> import copy
>>> a = [1, 2, 3]
>>> b = copy.copy(a)
>>> print(b)
```

But it is more idiomatic to use the slice operator.

8.11 Lists and strings

A string is a sequence of characters and a list is a sequence of values, but a list of characters is not the same as a string. To convert from a string to a list of characters, you can use the `list` function:

```
>>> s = 'spam'
>>> t = list(s)
>>> print(t)
['s', 'p', 'a', 'm']
```

`list` breaks a string into individual letters. If you want to break a string into words, you can use the `split` method:

```
>>> s = 'pining for the fjords'
>>> t = s.split()
>>> print(t)
['pining', 'for', 'the', 'fjords']
```

An optional argument called a **delimiter** specifies which characters to use as word boundaries. The following example uses `', '` (a comma followed by a space) as the delimiter:

```
>>> s = 'spam, spam, spam'
>>> delimiter = ', '
>>> s.split(delimiter)
['spam', 'spam', 'spam']
```

`join` is the inverse of `split`. It takes a list of strings and concatenates the elements. `join` is a string method, so you have to invoke it on the delimiter and pass the list as a parameter:

```
>>> t = ['pining', 'for', 'the', 'fjords']
>>> delimiter = ' '
>>> delimiter.join(t)
'pining for the fjords'
```

In this case the delimiter is a space character, so `join` puts a space between words. To concatenate strings without spaces, you can use the empty string, `''` as a delimiter.

Chapter 9

For Loops

9.1 Traversing a string

A lot of computations involve processing a string one character at a time. Often they start at the beginning, select each character in turn, do something to it, and continue until the end. This pattern of processing is called a **traversal**. One way to write a traversal is with a `while` statement:

```
index = 0
while index < len(fruit):
    letter = fruit[index]
    print(letter)
    index += 1
```

This loop traverses the string and displays each letter on a line by itself. The loop condition is `index < len(fruit)`, so when `index` is equal to the length of the string, the condition is false, and the body of the loop is not executed. The last character accessed is the one with the index `len(fruit)-1`, which is the last character in the string.
Exercise 9.1. *Write a function that takes a string as an argument and displays the letters backward, one per line.*

Another way to write a traversal is with a `for` loop:

```
for char in fruit:
    print(char)
```

Each time through the loop, the next character in the string is assigned to the variable `char`. The loop continues until no characters are left.

The following example shows how to use concatenation (string addition) and a `for` loop to generate an abecedarian series (that is, in alphabetical order). In Robert Mc-

Closkey's book *Make Way for Ducklings*, the names of the ducklings are Jack, Kack, Lack, Mack, Nack, Ouack, Pack, and Quack. This loop outputs these names in order:

```
prefixes = 'JKLMNOPQ'
suffix = 'ack'

for letter in prefixes:
    print(letter + suffix)
```

The output is:

```
Jack
Kack
Lack
Mack
Nack
Oack
Pack
Qack
```

Of course, that's not quite right because "Ouack" and "Quack" are misspelled.
Exercise 9.2. *Modify the program to fix this error.*

9.2 Traversing a list

The most common way to traverse the elements of a list is with a `for` loop. The syntax is the same as for strings:

```
for cheese in cheeses:
    print(cheese)
```

This works well if you only need to read the elements of the list. But if you want to write or update the elements, you need the indices. A common way to do that is to combine the functions `range` and `len`:

```
for i in range(len(numbers)):
    numbers[i] = numbers[i] * 2
```

This loop traverses the list and updates each element. `len` returns the number of elements in the list. `range` returns a list of indices from 0 to $n-1$, where n is the length of the list. Each time through the loop `i` gets the index of the next element. The assignment statement in the body uses `i` to read the old value of the element and to assign the new value.

`range` can also take more arguments. With two arguments, `range` returns a list that contains all the integers from the first to the second, including the first but not including the second! If there is a third argument, it specifies the space between successive values, which is called the "step size."

A `for` loop over an empty list never executes the body:

```
for x in empty:
    print('This never happens.')
```

Although a list can contain another list, the nested list still counts as a single element. The length of this list is four:

```
['spam!', 1, ['Brie', 'Roquefort', 'Pol le Veq'], [1, 2, 3]]
```

9.3 A `find` function

What does the following function do?

```
def find(word, letter):
    index = 0
    while index < len(word):
        if word[index] == letter:
            return index
        index = index + 1
    return -1
```

This use of loops is the basic logic behind the `find` method discussed earlier.

This is the first example we have seen of a `return` statement inside a loop. If `word[index] == letter`, the function breaks out of the loop and returns immediately.

If the character doesn't appear in the string, the program exits the loop normally and returns `-1`.

This pattern of computation—traversing a sequence and returning when we find what we are looking for—is a called a **search**.
Exercise 9.3. *Modify* `find` *so that it has a third parameter, the index in* `word` *where it should start looking.*

9.4 Looping and counting

The following program counts the number of times the letter `a` appears in a string:

```
word = 'banana'
count = 0
for letter in word:
    if letter == 'a':
        count = count + 1
print(count)
```

This program demonstrates another pattern of computation called a **counter**. The variable `count` is initialized to 0 and then incremented each time an `a` is found. When the loop exits, `count` contains the result—the total number of `a`'s.

Exercise 9.4. *Encapsulate this code in a function named* `count`*, and generalize it so that it accepts the string and the letter as arguments.*

Exercise 9.5. *Rewrite this function so that instead of traversing the string, it uses the three-parameter version of* `find` *from the previous section.*

9.5 The `in` operator

The operators we have seen so far are all special characters like `+` and `*`, but there are a few operators that are words. `in` is a boolean operator that takes two strings and returns `True` if the first appears as a substring in the second:

```
>>> 'an' in 'banana'
True
>>> 'c' in 'banana'
False
```

For example, the following function prints all the letters from `word1` that also appear in `word2`:

```
def in_both(word1, word2):
    for letter in word1:
        if letter in word2:
            print(letter)
```

With well-chosen variable names, Python sometimes reads like English. You could read this loop, "for (each) letter in (the first) word, if (the) letter (appears) in (the second) word, print (the) letter."

Here's what you get if you compare apples and oranges:

```
>>> in_both('apples', 'oranges')
a
e
s
```

The `in` operator also works on lists.

```
>>> cheeses = ['Cheddar', 'Edam', 'Gouda']
>>> 'Edam' in cheeses
True
```

```
>>> 'Brie' in cheeses
False
```

9.6 break

As with while loops, sometimes you don't know it's time to end a for loop until you get half way through the body. Since we can't reset a sentinal value to exit the loop, in this case we use the `break` statement to jump out of the loop.

For example, suppose you want to count the number of times the word ”lemur” appears in a list, but stop early if you see the word ”done”. You could write:

```
count = 0
animals = ['cat', 'lemur', 'fox', 'rabbit', 'lemur', 'done', 'lemur']
for item in animals:
    if item == 'lemur':
        count += 1
    elif item == 'done':
        break
print(count)
```

The loop now runs until either it consumes all elements of the list or it hits the break statement, and count will be equal to 2 after execution.

9.7 Map, filter and reduce

To add up all the numbers in a list, you can use a loop like this:

```
def add_all(t):
    total = 0
    for x in t:
        total += x
    return total
```

`total` is initialized to 0. Each time through the loop, `x` gets one element from the list. The += operator provides a short way to update a variable:

```
    total += x
```

is equivalent to:

```
    total = total + x
```

As the loop executes, `total` accumulates the sum of the elements; a variable used this way is sometimes called an **accumulator**.

Adding up the elements of a list is such a common operation that Python provides it as a built-in function, `sum`:

```
>>> t = [1, 2, 3]
>>> sum(t)
6
```

An operation like this that combines a sequence of elements into a single value is sometimes called **reduce**.

Sometimes you want to traverse one list while building another. For example, the following function takes a list of strings and returns a new list that contains capitalized strings:

```
def capitalize_all(t):
    res = []
    for s in t:
        res.append(s.capitalize())
    return res
```

`res` is initialized with an empty list; each time through the loop, we append the next element. So `res` is another kind of accumulator.

An operation like `capitalize_all` is sometimes called a **map** because it "maps" a function (in this case the method `capitalize`) onto each of the elements in a sequence.

Another common operation is to select some of the elements from a list and return a sublist. For example, the following function takes a list of strings and returns a list that contains only the uppercase strings:

```
def only_upper(t):
    res = []
    for s in t:
        if s.isupper():
            res.append(s)
    return res
```

`isupper` is a string method that returns `True` if the string contains only upper case letters.

An operation like `only_upper` is called a **filter** because it selects some of the elements and filters out the others.

Most common list operations can be expressed as a combination of map, filter and reduce. Because these operations are so common, Python provides language features to support them, including the built-in function `reduce` and an operator called a "list comprehension." But these features are idiomatic to Python, so I won't go into the details.

Exercise 9.6. *Write a function that takes a list of numbers and returns the cumulative sum; that is, a new list where the ith element is the sum of the first $i+1$ elements from the original list. For example, the cumulative sum of* `[1, 2, 3]` *is* `[1, 3, 6]`.

9.8 Debugging

When you are debugging a program, and especially if you are working on a hard bug, there are four things to try:

reading: Examine your code, read it back to yourself, and check that it means what you meant to say.

running: Experiment by making changes and running different versions. Often if you display the right thing at the right place in the program, the problem becomes obvious, but sometimes you have to spend some time to build scaffolding.

ruminating: Take some time to think! What kind of error is it: syntax, run-time, logical? What information can you get from the error messages, or from the output of the program? What kind of error could cause the problem you're seeing? What did you change last, before the problem appeared?

retreating: At some point, the best thing to do is back off, undoing recent changes, until you get back to a program that works, and that you understand. Then you can starting rebuilding.

Beginning programmers sometimes get stuck on one of these activities and forget the others. Each activity comes with its own failure mode.

For example, reading your code might help if the problem is a typographical error, but not if the problem is a conceptual misunderstanding. If you don't understand what your program does, you can read it 100 times and never see the error, because the error is in your head.

Running experiments can help, especially if you run small, simple tests. But if you run experiments without thinking or reading your code, you might fall into a pattern I call "random walk programming," which is the process of making random changes until the program does the right thing. Needless to say, random walk programming can take a long time.

The way out is to take more time to think. Debugging is like an experimental science. You should have at least one hypothesis about what the problem is. If there are two or more possibilities, try to think of a test that would eliminate one of them.

Taking a break sometimes helps with the thinking. So does talking. If you explain the problem to someone else (or even yourself), you will sometimes find the answer before you finish asking the question.

But even the best debugging techniques will fail if there are too many errors, or if the code you are trying to fix is too big and complicated. Sometimes the best option is to retreat, simplifying the program until you get to something that you understand, and that works.

Beginning programmers are often reluctant to retreat, because they can't stand to delete a line of code (even if it's wrong). If it makes you feel better, copy your program into another file before you start stripping it down. Then you can paste the pieces back in a little bit at a time.

To summarize, here's the Fifth Theorem of debugging:

> Finding a hard bug requires reading, running, ruminating, and sometimes retreating. If you get stuck on one of these activities, try the others.

9.9 Glossary

list: A sequence of values.

element: One of the values in a list (or other sequence), also called items.

index: An integer value that indicates an element in a list.

nested list: A list that is an element of another list.

list traversal: The sequential accessing of each element in a list.

mapping: A relationship in which each element of one set corresponds to an element of another set. For example, a list is a mapping from indices to elements.

accumulator: A variable used in a loop to add up or accumulate a result.

reduce: A processing pattern that traverses a sequence and accumulates the elements into a single result.

map: A processing pattern that traverses a sequence and performs an operation on each element.

filter: A processing pattern that traverses a list and selects the elements that satisfy some criterion.

object: Something a variable can refer to. An object has a type and a value.

equivalent: Having the same value.

identical: Being the same object (which implies equivalence).

reference: The association between a variable and its value.

aliasing: A circumstance where two variables refer to the same object.

delimiter: A character or string used to indicate where a string should be split.

9.10 Exercises

Exercise 9.7. *Write a function called* `is_sorted` *that takes a list as a parameter and returns* `True` *if the list is sorted in ascending order and* `False` *otherwise. You can assume (as a precondition) that the elements of the list can be compared with the comparison operators <, >, etc.*

For example, `is_sorted([1,2,2])` *should return* `True` *and* `is_sorted(['b','a'])` *should return* `False`.

Chapter 10

Recursion

10.1 Recursion

It is legal for one function to call another; it is also legal for a function to call itself. It may not be obvious why that is a good thing, but it turns out to be one of the most magical things a program can do. For example, look at the following function:

```
def countdown(n):
    if n <= 0:
        print('Blastoff!')
    else:
        print(n)
        countdown(n-1)
```

If `n` is 0 or negative, it outputs the word, "Blastoff!" Otherwise, it outputs `n` and then calls a function named `countdown`—itself—passing `n-1` as an argument.

What happens if we call this function like this?

```
>>> countdown(3)
```

The execution of `countdown` begins with `n=3`, and since `n` is greater than 0, it outputs the value 3, and then calls itself...

> The execution of `countdown` begins with `n=2`, and since `n` is greater than 0, it outputs the value 2, and then calls itself...
>
> > The execution of `countdown` begins with `n=1`, and since `n` is greater than 0, it outputs the value 1, and then calls itself...
> >
> > > The execution of `countdown` begins with `n=0`, and since `n` is not greater than 0, it outputs the word, "Blastoff!" and then returns.

The `countdown` that got `n=1` returns.

The `countdown` that got `n=2` returns.

The `countdown` that got `n=3` returns.

And then you're back in `__main__`. So, the total output looks like this:

```
3
2
1
Blastoff!
```

A function that calls itself is **recursive**; the process is called **recursion**.

As another example, we can write a function that prints a string `n` times.

```
def print_n(s, n):
    if n <= 0:
        return
    print(s)
    print_n(s, n-1)
```

If `n <= 0` the `return` statement exits the function. The flow of execution immediately returns to the caller, and the remaining lines of the function are not executed.

The rest of the function is similar to `countdown`: if `n` is greater than 0, it displays `s` and then calls itself to display `s` $n-1$ additional times. So the number of lines of output is `1 + (n - 1)` which, if you do your algebra right, comes out to `n`.

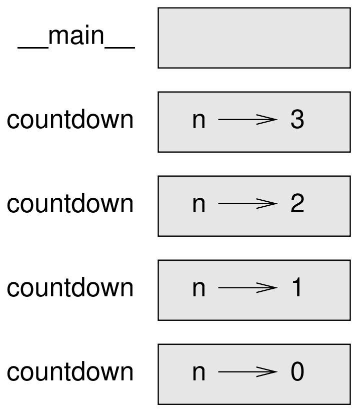
For simple examples like this, it is probably easier to use a `for` loop. But we will see examples later that are hard to write with a `for` loop and easy to write with recursion, so it is good to start early.

10.2 Stack diagrams for recursive functions

In Section 5.7, we used a stack diagram to represent the state of a program during a function call. The same kind of diagram can help interpret a recursive function.

Every time a function gets called, Python creates a new function frame, which contains the function's local variables and parameters. For a recursive function, there might be more than one frame on the stack at the same time.

This figure shows a stack diagram for `countdown` called with `n = 3`:



As usual, the top of the stack is the frame for `__main__`. It is empty because we did not create any variables in `__main__` or pass any arguments to it.

The four `countdown` frames have different values for the parameter `n`. The bottom of the stack, where `n=0`, is called the **base case**. It does not make a recursive call, so there are no more frames.

Draw a stack diagram for `print_n` called with `s = 'Hello'` and `n=4`.

10.3 Infinite recursion

If a recursion never reaches a base case, it goes on making recursive calls forever, and the program never terminates. This is known as **infinite recursion**, and it is generally not a good idea. Here is a minimal program with an infinite recursion:

```
def recurse():
    recurse()
```

In most programming environments, a program with infinite recursion does not really run forever. Python reports an error message when the maximum recursion depth is reached:

```
  File "<stdin>", line 2, in recurse
  File "<stdin>", line 2, in recurse
  File "<stdin>", line 2, in recurse
                  .
                  .
                  .
  File "<stdin>", line 2, in recurse
RuntimeError: Maximum recursion depth exceeded
```

This traceback is a little bigger than the one we saw in the previous chapter. When the error occurs, there are 1000 `recurse` frames on the stack!

10.4 More recursion

We have only covered a small subset of Python, but you might be interested to know that this subset is a *complete* programming language, which means that anything that can be computed can be expressed in this language. Any program ever written could be rewritten using only the language features you have learned so far (actually, you would need a few commands to control devices like the keyboard, mouse, disks, etc., but that's all).

Proving that claim is a nontrivial exercise first accomplished by Alan Turing, one of the first computer scientists (some would argue that he was a mathematician, but a lot of early computer scientists started as mathematicians). Accordingly, it is known as the Turing Thesis. If you take a course on the Theory of Computation, you will have a chance to see the proof.

To give you an idea of what you can do with the tools you have learned so far, we'll evaluate a few recursively defined mathematical functions. A recursive definition is similar to a circular definition, in the sense that the definition contains a reference to the thing being defined. A truly circular definition is not very useful:

frabjuous: An adjective used to describe something that is frabjuous.

If you saw that definition in the dictionary, you might be annoyed. On the other hand, if you looked up the definition of the factorial function, denoted with the symbol !, you might get something like this:

$$0! = 1$$
$$n! = n(n-1)!$$

This definition says that the factorial of 0 is 1, and the factorial of any other value, n, is n multiplied by the factorial of $n-1$.

So 3! is 3 times 2!, which is 2 times 1!, which is 1 times 0!. Putting it all together, 3! equals 3 times 2 times 1 times 1, which is 6.

If you can write a recursive definition of something, you can usually write a Python program to evaluate it. The first step is to decide what the parameters should be. In this case it should be clear that `factorial` has a single parameter:

```
def factorial(n):
```

If the argument happens to be 0, all we have to do is return 1:

```
def factorial(n):
    if n == 0:
        return 1
```

Otherwise, and this is the interesting part, we have to make a recursive call to find the factorial of $n-1$ and then multiply it by n:

```
def factorial(n):
    if n == 0:
        return 1
    else:
        recurse = factorial(n-1)
        result = n * recurse
        return result
```

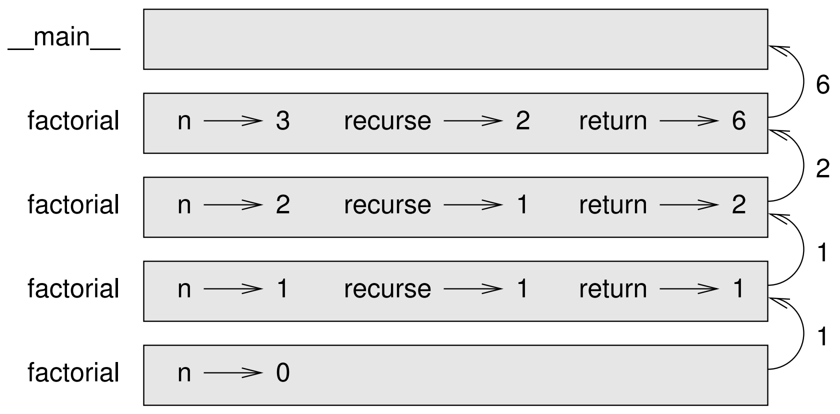
The flow of execution for this program is similar to the flow of `countdown` in Section 10.1. If we call `factorial` with the value 3:

Since 3 is not 0, we take the second branch and calculate the factorial of `n-1`...

> Since 2 is not 0, we take the second branch and calculate the factorial of `n-1`...
>
> > Since 1 is not 0, we take the second branch and calculate the factorial of `n-1`...
> >
> > > Since 0 *is* 0, we take the first branch and return 1 without making any more recursive calls.
> >
> > The return value (1) is multiplied by *n*, which is 1, and the result is returned.
>
> The return value (1) is multiplied by *n*, which is 2, and the result is returned.

The return value (2) is multiplied by *n*, which is 3, and the result, 6, becomes the return value of the function call that started the whole process.

Here is what the stack diagram looks like for this sequence of function calls:



The return values are shown being passed back up the stack. In each frame, the return value is the value of `result`, which is the product of `n` and `recurse`.

In the last frame, the local variables `recurse` and `result` do not exist, because the branch that creates them did not execute.

10.5 Leap of faith

Following the flow of execution is one way to read programs, but it can quickly become labyrinthine. An alternative is what I call the "leap of faith." When you come to a function call, instead of following the flow of execution, you *assume* that the function works correctly and returns the right result.

In fact, you are already practicing this leap of faith when you use built-in functions. When you call `math.cos` or `math.exp`, you don't examine the bodies of those functions. You just assume that they work because the people who wrote the built-in functions were good programmers.

The same is true when you call one of your own functions. For example, in Section 5.10, we wrote a function called `is_divisible` that determines whether one number is divisible by another. Once we have convinced ourselves that this function is correct—examining the code and testing—we can use the function without looking at the code again.

The same is true of recursive programs. When you get to the recursive call, instead of following the flow of execution, you should assume that the recursive call works (yields the correct result) and then ask yourself, "Assuming that I can find the factorial of $n-1$, can I compute the factorial of n?" In this case, it is clear that you can, by multiplying by n.

Of course, it's a bit strange to assume that the function works correctly when you haven't finished writing it, but that's why it's called a leap of faith!

10.6 One more example

After `factorial`, the most common example of a recursively defined mathematical function is `fibonacci`, which has the following definition:

$$\begin{aligned}
\text{fibonacci}(0) &= 0 \\
\text{fibonacci}(1) &= 1 \\
\text{fibonacci}(n) &= \text{fibonacci}(n-1) + \text{fibonacci}(n-2);
\end{aligned}$$

Translated into Python, it looks like this:

```
def fibonacci (n):
    if n == 0:
        return 0
    elif  n == 1:
        return 1
    else:
        return fibonacci(n-1) + fibonacci(n-2)
```

If you try to follow the flow of execution here, even for fairly small values of *n*, your head explodes. But according to the leap of faith, if you assume that the two recursive calls work correctly, then it is clear that you get the right result by adding them together.

10.7 Checking types

What happens if we call `factorial` and give it 1.5 as an argument?

```
>>> factorial(1.5)
RuntimeError: Maximum recursion depth exceeded
```

It looks like an infinite recursion. But how can that be? There is a base case—when `n == 0`. But if `n` is not an integer, we can *miss* the base case and recurse forever.

In the first recursive call, the value of `n` is 0.5. In the next, it is -0.5. From there, it gets smaller and smaller, but it will never be 0.

We have two choices. We can try to generalize the `factorial` function to work with floating-point numbers, or we can make `factorial` check the type of its argument. The first option is called the gamma function and it's a little beyond the scope of this book. So we'll go for the second.

We can use the built-in function `isinstance` to verify the class of the argument. While we're at it, we can also make sure the argument is positive:

```
def factorial (n):
    if not isinstance(n, int):
        print('Factorial is only defined for integers.')
        return None
    elif n < 0:
        print('Factorial is only defined for positive integers.')
        return None
    elif n == 0:
        return 1
    else:
        return n * factorial(n-1)
```

Now we have three base cases. The first catches nonintegers and the second catches negative integers. In both cases, the program prints an error message and returns `None` to indicate that something went wrong:

```
>>> factorial('fred')
Factorial is only defined for integers.
None
>>> factorial(-2)
Factorial is only defined for positive integers.
None
```

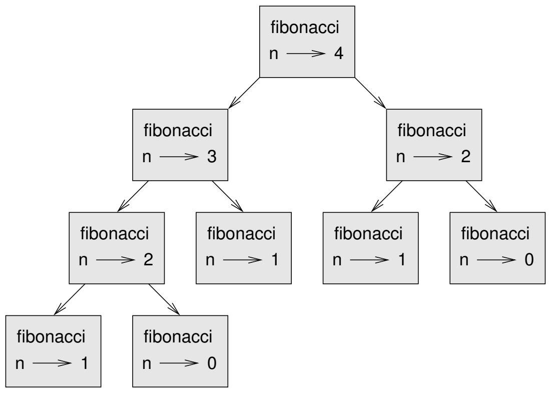
If we get past both checks, then we know that *n* is a positive integer, and we can prove that the recursion terminates.

This program demonstrates a pattern sometimes called a **guardian**. The first two conditionals act as guardians, protecting the code that follows from values that might cause an error. The guardians make it possible to prove the correctness of the code.

10.8 Hints

If you played with the `fibonacci` function from Section 10.6, you might have noticed that the bigger the argument you provide, the longer the function takes to run. Furthermore, the run time increases very quickly.

To understand why, consider this **call graph** for `fibonacci` with `n=4`:



A call graph shows a set function frames, with lines connecting each frame to the frames of the functions it calls. At the top of the graph, `fibonacci` with `n=4` calls `fibonacci` with `n=3` and `n=2`. In turn, `fibonacci` with `n=3` calls `fibonacci` with `n=2` and `n=1`. And so on.

Count how many times `fibonacci(0)` and `fibonacci(1)` are called. This is an inefficient solution to the problem, and it gets worse as the argument gets bigger.

One solution is to keep track of values that have already been computed by storing them in a list. A previously computed value that is stored for later use is called a **hint**. Here is an implementation of `fibonacci` using hints:

```
previous = [0, 1]

def fibonacci(n):
    if n < len(previous):
        return previous[n]

    res = fibonacci(n-1) + fibonacci(n-2)
    previous.append(res)
    return res
```

`previous` keeps track of the Fibonacci numbers we already know. We start with only two items: 0 and 1.

Whenever `fibonacci` is called, it checks `previous`. If the result is already there, it can return immediately. Otherwise it has to compute the new value, append it to the previous list, and return it.

`previous` is created outside the function, so it belongs to the special frame called `__main__`. Variables in `__main__` are sometimes called **global** because they can be accessed from any function. Unlike local variables, which disappear when their function ends, global variables persist from one function call to the next.

Using this version of `fibonacci`, you can compute `fibonacci(40)` in an eyeblink. If you compute `fibonacci(50)`, you get:

```
>>> fibonacci(50)
12586269025
```

10.9 Debugging

Breaking a large program into smaller functions creates natural checkpoints for debugging. If a function is not working, there are three possibilities to consider:

- There is something wrong with the arguments the function is getting.
- There is something wrong with the function.
- There is something wrong with the return value or the way it is being used.

To rule out the first possibility, you can add a `print` statement at the beginning of the function and display the values of the parameters (and maybe their types).

If the parameters look good, add a `print` statement before each `return` statement that displays the return value. If possible, check the result by hand. If necessary, call the function with special values where you know what the result should be (as in Section 5.11).

If the function seems to be working, look at the function call to make sure the return value is being used correctly (or used at all!).

Adding print functions at the beginning and end of a function can help make the flow of execution more visible. For example, here is a version of `factorial` with print functions:

```
def factorial(n):
    space = ' ' * (4 * n)
    print(space, 'factorial', n)
    if n == 0:
        print(space, 'returning 1')
        return 1
    else:
        recurse = factorial(n-1)
        result = n * recurse
        print(space, 'returning', result)
        return result
```

`space` is a string of space characters that controls the indentation of the output. Here is the result of `factorial(5)` :

```
                     factorial 5
                 factorial 4
             factorial 3
         factorial 2
     factorial 1
 factorial 0
 returning 1
     returning 1
         returning 2
             returning 6
                 returning 24
                     returning 120
```

If you are confused about the flow of execution, this kind of output can be helpful. It takes some time to develop effective scaffolding, but according to the Sixth Theorem of Debugging:

> A little bit of scaffolding can save a lot of debugging.

10.10 Glossary

recursion: The process of calling the function that is currently executing.

base case: A conditional branch in a recursive function that does not make a recursive call.

infinite recursion: A function that calls itself recursively without ever reaching the base case. Eventually, an infinite recursion causes a runtime error.

10.11 Exercises

Exercise 10.1. *Draw a stack diagram for the following program. What does the program print?*

```
def b(z):
    prod = a(z, z)
    print(z, prod)
    return prod

def a(x, y):
    x = x + 1
    return x * y

def c(x, y, z):
    sum = x + y + z
    pow = b(sum)**2
    return pow

x = 1
y = x + 1
print(c(x, y+3, x+y))
```

Chapter 11

Files

11.1 Persistence

Most of the programs we have seen so far are transient in the sense that they run for a short time and produce some output, but when they end, their data disappears. If you run the program again, it starts with a clean slate.

Other programs are **persistent**: they run for a long time (or all the time); they keep at least some of their data in non-volatile storage (a hard drive, for example); and if they shut down and restart, they pick up where they left off.

Examples of persistent programs are operating systems, which run pretty much whenever a computer is on, and web servers, which run all the time, waiting for requests to come in on the network.

One of the simplest ways for programs to maintain their data is by reading and writing text files. We have already seen programs that read text files; in this chapters we will see programs that write them.

An alternative is to store the state of the program in a database. In this chapter I will present a simple database and a module, `pickle`, that makes it easy to store program data.

11.2 Reading and writing

A text file is a sequence of characters stored on a permanent medium like a hard drive, flash memory, or CD-ROM. To read a file, you can use `open` to create a file object:

```
>>> fin = open('words.txt')
>>> print(fin)
<open file 'words.txt', mode 'r' at 0xb7eb2380>
```

Mode `'r'` means that this file is open for reading. The file object provides several methods for reading data, including `readline`:

```
>>> line = fin.readline()
>>> print(line)
aa
```

The file object keeps track of where it is in the file, so if you invoke `readline` again, it picks up from where it left off. You can also use a file object in a for loop.

To write a file, you have to create a file object with mode `'w'` as a second parameter:

```
>>> fout = open('output.txt', 'w')
>>> print(fout)
<open file 'output.txt', mode 'w' at 0xb7eb2410>
```

If the file already exists, opening it in write mode clears out the old data and starts fresh, so be careful! If the file doesn't exist, a new one is created.

The `write` method puts data into the file.

```
>>> line1 = "This here's the wattle,\n"
>>> fout.write(line1)
```

Again, the file object keeps track of where it is, so if you call `write` again, it add the new data to the end.

```
>>> line2 = "the emblem of our land.\n"
>>> fout.write(line2)
```

When you are done writing, you have to close the file.

```
>>> fout.close()
```

11.3 Format operator

The argument of `write` has to be a string, so if we want to put other values in a file, we have to convert them to strings. The easiest way to do that is with `str`:

```
>>> x = 52
>>> f.write(str(x))
```

An alternative is to use the **format operator**, `%`. When applied to integers, `%` is the modulus operator. But when the first operand is a string, `%` is the format operator.

The first operand is the **format string**, and the second operand is a tuple of expressions. The result is a string that contains the values of the expressions, formatted according to the format string.

As an example, the **format sequence** '%d' means that the first expression in the tuple should be formatted as an integer (d stands for "decimal"):

```
>>> camels = 42
>>> '%d' % camels
'42'
```

The result is the string '42', which is not to be confused with the integer value 42.

A format sequence can appear anywhere in the format string, so you can embed a value in a sentence:

```
>>> camels = 42
>>> 'I have spotted %d camels.' % camels
'I have spotted 42 camels.'
```

The format sequence '%g' formats the next element in the tuple as a floating-point number (don't ask why), and '%s' formats the next item as a string:

```
>>> 'In %d years I have spotted %g %s.' % (3, 0.1, 'camels')
'In 3 years I have spotted 0.1 camels.'
```

By default, the floating-point format prints six decimal places.

The number of elements in the tuple has to match the number of format sequences in the string. Also, the types of the elements have to match the format sequences:

```
>>> '%d %d %d' % (1, 2)
TypeError: not enough arguments for format string
>>> '%d' % 'dollars'
TypeError: illegal argument type for built-in operation
```

In the first example, there aren't enough elements; in the second, the element is the wrong type.

You can specify the number of digits as part of the format sequence. For example, the sequence '%8.2f' formats a floating-point number to be 8 characters long, with 2 digits after the decimal point:

```
>>> '%8.2f' % 3.14159
'    3.14'
```

The result takes up eight spaces with two digits after the decimal point.

11.4 Filenames and paths

Files are organized into **directories** (also called "folders"). Every running program has a "current directory," which is the default directory for most operations. For example, when you create a new file with `open`, the new file goes in the current directory. And when you open a file for reading, Python looks for it in the current directory.

The module `os` provides functions for working with files and directories ("os" stands for "operating system"). `os.getcwd` returns the name of the current directory:

```
>>> import os
>>> cwd = os.getcwd()
>>> print(cwd)
/home/dinsdale
```

`cwd` stands for "current working directory." The result in this example is `/home/dinsdale`, which is the home directory of a user named `dinsdale`.

A string like `cwd` that identifies a file is called a **path**. A **relative path** starts from the current directory; an **absolute path** starts from the topmost directory in the file system.

The paths we have seen so far are simple filenames, so they are relative to the current directory. To find the absolute path to a file, you can use `abspath`, which is in the module `os.path`.

```
>>> os.path.abspath('memo.txt')
'/home/dinsdale/memo.txt'
```

`os.path.exists` checks whether the file (or directory) specified by a path exists:

```
>>> os.path.exists('memo.txt')
True
```

If it exists, `os.path.isdir` checks whether it's a directory:

```
>>> os.path.isdir('memo.txt')
False
>>> os.path.isdir('music')
True
```

Similarly, `os.path.isfile` checks whether it's a file.

`os.listdir` returns a list of the files (and other directories) in the given directory:

```
>>> os.listdir(cwd)
['music', 'photos', 'memo.txt']
```

To demonstrate these functions, the following example "walks" through a directory, prints the names of all the files, and calls itself recursively on all the directories.

```
def walk(dir):
    for name in os.listdir(dir):
        path = os.path.join(dir, name)
        if os.path.isfile(path):
            print(path)
        else:
            walk(path)
```

`os.path.join` takes a directory and a file name and joins them into a complete path.
Exercise 11.1. *Modify* `walk` *so that instead of printing the names of the files, it returns a list of names.*

11.5 Catching exceptions

A lot of things can go wrong when you try to read and write files. If you try to open a file that doesn't exist, you get an `IOError`:

```
>>> fin = open('bad_file')
IOError: [Errno 2] No such file or directory: 'bad_file'
```

If you don't have permission to access a file:

```
>>> fout = open('/etc/passwd', 'w')
IOError: [Errno 13] Permission denied: '/etc/passwd'
```

And if you try to open a directory for reading, you get

```
>>> fin = open('/home')
IOError: [Errno 21] Is a directory
```

To avoid these errors, you could use functions like `os.path.exists` and `os.path.isfile`, but it would take a lot of time and code to check all the possibilities (based on the last error message, there are *at least* 21 things that can go wrong).

It is better to go ahead and try, and deal with problems if they happen, which is exactly what the `try` statement does. The syntax is similar to an `if` statement:

```
try:
    fin = open('bad_file')
    for line in fin:
        print(line)
    fin.close()
except:
    print('Something went wrong.')
```

Python starts by executing the `try` clause. If all goes well, it skips the `except` clause and proceeds. If an exception occurs, it jumps out of the `try` clause and executes the `except` clause.

Handling an exception with a `try` statement is called **catching** an exception. In this example, the `except` clause prints an error message that is not very helpful. In general, catching an exception gives you a chance to fix the problem, or try again, or at least end the program gracefully.

11.6 Pickling

If you want to store the state of a program, and not just strings to a file, the `pickle` module can help. It translates almost any type of object into a string, suitable for storage in a database, and then translates strings back into objects.

`pickle.dumps` takes an object as a parameter and returns a string representation (`dumps` is short for "dump string"):

```
>>> import pickle
>>> t = [1, 2, 3]
>>> pickle.dumps(t)
b'\x80\x03]q\x00(K\x01K\x02K\x03e.'
```

The format isn't obvious to human readers; it is meant to be easy for `pickle` to interpret. `pickle.loads` ("load string") reconstitutes the object:

```
>>> t1 = [1, 2, 3]
>>> s = pickle.dumps(t1)
>>> t2 = pickle.loads(s)
>>> print(t2)
[1, 2, 3]
```

Although the new object has the same value as the old, it is not (in general) the same object:

```
>>> t == t2
True
>>> t is t2
False
```

In other words, pickling and then unpickling has the same effect as copying the object.

You can use `pickle` to store non-strings in a database. In fact, this combination is so common that it has been encapsulated in a module called `shelve`.

11.7 Glossary

persistent: Pertaining to a program that runs indefinitely and keeps at least some of its data in permanent storage.

format operator: An operator, `%`, that takes a format string and a tuple and generates a string that includes the elements of the tuple formatted as specified by the format string.

format string: A string, used with the format operator, that contains format sequences.

format sequence: A sequence of characters in a format string, like `%d` that specifies how a value should be formatted.

text file: A sequence of characters stored in non-volatile storage like a hard drive.

directory: A named collection of files, also called a folder.

path: A string that identifies a file.

relative path: A path that starts from the current directory.

absolute path: A path that starts from the topmost directory in the file system.

catch: To prevent an exception from terminating a program using the `try` and `except` statements.

Chapter 12

Dictionaries

A **dictionary** is like a list, but more general. In a list, the indices have to be integers; in a dictionary they can be (almost) any type.

You can think of a dictionary as a mapping between a set of indices and a set of values. Each index, which is called a **key**, corresponds to a value. The association of a key and a value is called a **key-value pair** or sometimes an **item**.

As an example, we will build a dictionary that maps from English words to Spanish words, so the keys and the values are all strings.

The function `dict` creates a new dictionary with no items.

```
>>> eng2sp = dict()
>>> print(eng2sp)
{}
```

The squiggly-brackets, `{}`, represent an empty dictionary. To add items to the dictionary, you can use square brackets:

```
>>> eng2sp['one'] = 'uno'
```

This line creates an item that maps from the key `'one'` to the value `'uno'`. If we print the dictionary again, we see a key-value pair with a colon between the key and value:

```
>>> print(eng2sp)
{'one': 'uno'}
```

This output format is also an input format. For example, you can create a new dictionary with three items:

```
>>> eng2sp = {'one': 'uno', 'two': 'dos', 'three': 'tres'}
```

But if you print `eng2sp`, you might be surprised:

```
>>> print(eng2sp)
{'one': 'uno', 'three': 'tres', 'two': 'dos'}
```

The key-value pairs are not in order, but that's not a problem because the elements of a dictionary are never indexed with integer indices. Instead, you use the keys to look up the corresponding values:

```
>>> print(eng2sp['two'])
'dos'
```

The key `'two'` always maps to the value `'dos'` so the order of the items doesn't matter.

If the key isn't in the dictionary, you get an exception:

```
>>> print(eng2sp['four'])
KeyError: 'four'
```

The `len` function works on dictionaries; it returns the number of key-value pairs:

```
>>> len(eng2sp)
3
```

The `in` operator works on dictionaries; it tells you whether something appears as a *key* in the dictionary (appearing as a value is not good enough).

```
>>> 'one' in eng2sp
True
>>> 'uno' in eng2sp
False
```

To see whether something appears as a value in a dictionary, you can use the method `values`, and then use the `in` operator:

```
>>> vals = eng2sp.values()
>>> 'uno' in vals
True
```

The `in` operator uses different algorithms for lists and dictionaries. For lists, it uses a search algorithm, as in Section 9.3. As the list gets longer, the search time gets longer in direct proportion. For dictionaries, Python uses an algorithm called a **hashtable** that has a remarkable property: the `in` operator takes about the same amount of time no matter how many items there are in a dictionary. I won't explain how that's possible, but you can look it up.

12.1 Dictionary as a set of counters

Suppose you are given a string and you want to count how many times each letter appears. There are several ways you could do it:

1. You could create 26 variables, one for each letter of the alphabet. Then you could traverse the string and, for each character, increment the corresponding counter, probably using a chained conditional.

2. You could create a list with 26 elements. Then you could convert each character to a number (using the built-in function `ord`), use the number as an index into the list, and increment the appropriate counter.

3. You could create a dictionary with characters as keys and counters as the corresponding values. The first time you see a character, you would add an item to the dictionary. After that you would increment the value of an existing item.

Each of these options performs the same computation, but each of them implements that computation in a different way.

An **implementation** is a way of performing a computation; some implementations are better than others. For example, an advantage of the dictionary implementation is that we don't have to know ahead of time which letters appear in the string and we only have to make room for the letters that do appear.

Here is what the code might look like:

```
def histogram(s):
    d = {}
    for c in s:
        if c not in d:
            d[c] = 1
        else:
            d[c] += 1
    return d
```

The name of the function is **histogram**, which is a statistical term for a set of counters (or frequencies).

The first line of the function creates an empty dictionary. The `for` loop traverses the string. Each time through the loop, if the character `c` is not in the dictionary, we create a new item with key `c` and the initial value 1 (since we have seen this letter once). If `c` is already in the dictionary we increment `d[c]`.

Here's how it works:

```
>>> h = histogram('brontosaurus')
>>> print(h)
{'a': 1, 'b': 1, 'o': 2, 'n': 1, 's': 2, 'r': 2, 'u': 2, 't': 1}
```

The histogram indicates that the letters `'a'` and `'b'` appear once each; `'o'` appears twice, and so on.

Exercise 12.1. *Dictionaries have a method called* `get` *that takes a key and a default value. If the key appears in the dictionary,* `get` *returns the corresponding value; otherwise it returns the default value. For example:*

```
>>> h = histogram('a')
>>> print(h)
{'a': 1}
>>> h.get('a', 0)
1
>>> h.get('b', 0)
0
```

Use `get` *to write* `histogram` *more concisely. You should be able to eliminate the* `if` *statement.*

12.2 Looping and dictionaries

If you use a dictionary in a `for` statement, it traverses the keys of the dictionary. For example, `print_hist` prints each key and the corresponding value:

```
def print_hist(h):
    for c in h:
        print(c, h[c])
```

Here's what the output looks like:

```
>>> h = histogram('parrot')
>>> print_hist(h)
a 1
p 1
r 2
t 1
o 1
```

Again, the keys are in no particular order.
Exercise 12.2. *Dictionaries have a method called* `keys` *that returns the keys of the dictionary, in no particular order, as a list.*

Modify `print_hist` *to print the keys and their values in alphabetical order, using* `keys` *and* `sort`.

12.3 Reverse lookup

Given a dictionary `d` and a key `k`, it is easy to find the corresponding value `v = d[k]`. This operation is called a **lookup**.

But what if you have v and you want to find k? You have two problems: first, there might be more than one key that maps to the value v. Depending on the application, you might be able to pick one, or you might have to make a list that contains all of them. Second, there is no simple syntax to do a **reverse lookup**; you have to search.

Here is a function that takes a value and returns the first key that maps to that value:

```
def reverse_lookup(d, v):
    for k in d:
        if d[k] == v:
            return k
    raise ValueError
```

This function is yet another example of the search pattern we have seen before, but it uses a feature we haven't seen before, raise. The raise statement causes an exception; in this case it causes a ValueError, which generally indicates that there is something wrong with the value of a parameter.

If we get to the end of the loop, that means v doesn't appear in the dictionary as a value, so we raise an exception.

Here is an example of a successful reverse lookup:

```
>>> h = histogram('parrot')
>>> k = reverse_lookup(h, 2)
>>> print(k)
r
```

And an unsuccessful one:

```
>>> k = reverse_lookup(h, 3)
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
  File "<stdin>", line 5, in reverse_lookup
ValueError
```

The result when you raise an exception is the same as when Python raises one: it prints a traceback and an error message.

The raise statement takes a detailed error message as an optional argument. For example:

```
>>> raise ValueError, 'value does not appear in the dictionary'
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
ValueError: value does not appear in the dictionary
```

A reverse lookup is much slower than a forward lookup; if you have to do it often, or if the dictionary gets big, the performance of your program will suffer.
Exercise 12.3. *Modify* reverse_lookup *so that it builds and returns a list of* all *keys that map to* v*, or an empty list if there are none.*

12.4 Dictionaries and lists

Lists can appear as values in a dictionary. For example, if you were given a dictionary that maps from letters to frequencies, you might want to invert it; that is, create a dictionary that maps from frequencies to letters. Since there might be several letters with the same frequency, each value in the inverted dictionary should be a list of letters.

Here is a function that inverts a dictionary:

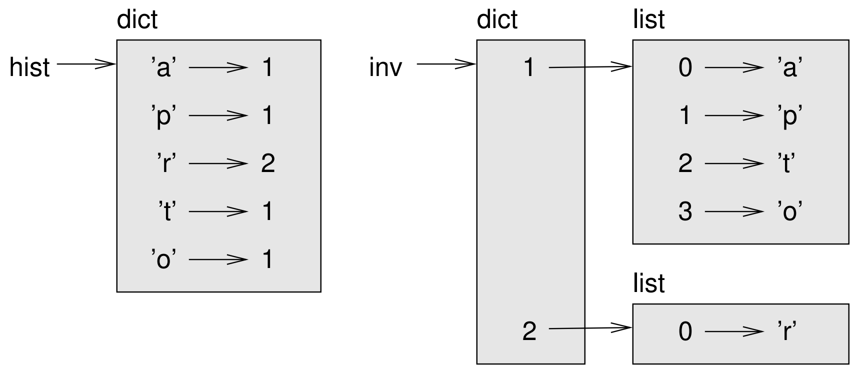
```
def invert_dict(d):
    inv = {}
    for key in d:
        val = d[key]
        if val not in inv:
            inv[val] = [key]
        else:
            inv[val].append(key)
    return inv
```

Each time through the loop, `key` gets a key from `d` and `val` gets the corresponding value. If `val` is not in `inv`, that means we haven't seen it before, so we create a new item and initialize it with a **singleton** (a list that contains a single element). Otherwise we have seen this value before, so we append the corresponding key to the list.

Here is an example:

```
>>> hist = histogram('parrot')
>>> print(hist)
{'a': 1, 'p': 1, 'r': 2, 't': 1, 'o': 1}
>>> inv = invert_dict(hist)
>>> print(inv)
{1: ['a', 'p', 't', 'o'], 2: ['r']}
```

And here is a diagram showing `hist` and `inv`:



A dictionary is represented as a box with the type `dict` above it and the key-value pairs inside. If the values are integers, floats or strings, I usually draw them inside the box, but I usually draw lists outside the box, just to keep the diagram simple.

Lists can be values in a dictionary, as this example shows, but they cannot be keys. Here's what happens if you try:

```
>>> t = [1, 2, 3]
>>> d = {}
>>> d[t] = 'oops'
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
TypeError: list objects are unhashable
```

I mentioned earlier that a dictionary is implemented using a hashtable and that means that the keys have to be **hashable**.

A **hash** is a function that takes a value (of any kind) and returns an integer. Dictionaries uses these integers, called hash values, to store and look up key-value pairs.

This system works fine if the keys are immutable. But if the keys are mutable, like lists, bad things happen. For example, when you create a key-value pair, Python hashes the key and stores it in the corresponding location. If you modify the key and then hash it again, it would go to a different location. In that case you might have two entries for the same key, or you might not be able to find a key. Either way, the dictionary wouldn't work correctly.

That's why the keys have to be hashable, and why mutable types like lists aren't. The simplest way to get around this limitation is to use tuples.

Since dictionaries are mutable, they can't be used as keys, but they *can* be used as values.

Exercise 12.4. *Read the documentation of the dictionary method* `setdefault` *and use it to write a more concise version of* `invert_dict`.

12.5 Debugging

As you work with bigger datasets it can become unwieldy to debug by printing and checking data by hand. Here are some suggestions for debugging large datasets:

Scale down the input: If possible, reduce the size of the dataset. For example if the program reads a text file, start with just the first 10 lines, or with the smallest example you can find. You can either edit the files themselves, or (better) modify the program so it reads only the first `n` lines.

If there is an error, you can reduce `n` to the smallest value that manifests the error, and then increase it gradually as you find and correct errors.

Check summaries and types: Instead of printing and checking the entire dataset, consider printing summaries of the data: for example, the number of items in a dictionary or the total of a list of numbers.

A common cause of run-time errors is a value that is not the right type. For debugging this kind of error, it is often enough to print the type of a value, which is often smaller than the value itself.

Write self-checks: Sometimes you can write code to check for errors automatically. For example, if you are computing the average of a list of number, you could check that the result is not greater than the largest element in the list or less than the smallest. This is called a "sanity check" because it detects results that are "insane."

Another kind of check compares the results of two different computations to see if they are consistent. This is called a "consistency check."

12.6 Glossary

dictionary: A mapping from a set of keys to their corresponding values.

key-value pair: The representation of the mapping from a key to a value.

item: Another name for a key-value pair.

key: An object that appears in a dictionary as the first part of a key-value pair.

value: An object that appears in a dictionary as the second part of a key-value pair. This is more specific than our previous use of the word "value."

implementation: A way of performing a computation.

hashtable: The algorithm used to implement Python dictionaries.

hash function: A function used by a hashtable to compute the location for a key.

hashable: A type that has a hash function. Immutable types like integers, floats and strings are hashable; mutable types like lists and dictionaries are not.

lookup: A dictionary operation that takes a key and finds the corresponding value.

reverse lookup: A dictionary operation that takes a value and finds one or more keys that map to it.

singleton: A list (or other sequence) with a single element.

call graph: A diagram that shows every frame created during the execution of a program, with an arrow from each caller to each callee.

histogram: A set of counters.

hint: A computed value stored to avoid unnecessary future computation.

global variable: A variable defined outside a function. Global variables can be accessed from any function.

12.7 Exercises

Exercise 12.5. *Two words are anagrams if you can rearrange the letters from one to spell the other. Write a function called* `is_anagram` *that takes two strings and returns True if they are anagrams.*

Exercise 12.6. *Write a function named* `has_duplicates` *that takes a list as a parameter and that returns* `True` *if there is any object that appears more than once in the list, and* `False` *otherwise.*

Part III

Object-Oriented Programming

Chapter 13

Classes and objects

13.1 User-defined types

We have used many of Python's built-in types; now we are going to define a new type. As an example, we will create a type called `Point` that represents a point in two-dimensional space.

In mathematical notation, points are often written in parentheses with a comma separating the coordinates. For example, $(0,0)$ represents the origin, and (x,y) represents the point x units to the right and y units up from the origin.

There are several ways we might represent points in Python:

- We could store the coordinates separately in two variables, `x` and `y`.
- We could store the coordinates as elements in a list or tuple.
- We could create a new type to represent points as objects.

Creating a new type is (a little) more complicated than the other options, but it has advantages that will be apparent soon.

A user-defined type is also called a **class**. A class definition looks like this:

```
class Point:
    """represents a point in 2-D space"""
```

This header indicates that the new class is called `Point`. The body is a docstring that explains what the class is for. You can define variables and functions inside a class definition, but we will get back to that later.

Defining a class named `Point` creates a class object, also named `Point`.

```
>>> print(Point)
<class __main__.Point>
>>> type(Point)
<class 'type'>
```

Because `Point` is defined at the top level, its "full name" is `__main__.Point`.

The class object is like a factory for creating objects. To create a Point, you call `Point` as if it were a function.

```
>>> blank = Point()
>>> print(blank)
<__main__.Point instance at 0xb7e9d3ac>
```

The return value is a reference to a Point object, which we assign to `blank`. Creating a new object is called **instantiation**, and the object is an **instance** of the class.

13.2 Attributes

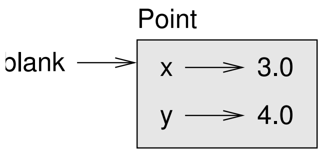
You can assign values to an instance using dot notation:

```
>>> blank.x = 3.0
>>> blank.y = 4.0
```

This syntax is similar to the syntax for selecting a variable from a module, such as `math.pi` or `string.uppercase`. In this case, though, we are assigning values to named elements of an object. These elements are called **attributes**.

As a noun, "AT-trib-ute" is pronounced with emphasis on the first syllable, as opposed to "a-TRIB-ute," which is a verb.

The following diagram shows the result of these assignments. A state diagram that shows an object and its attributes is called an **object diagram**:



The variable `blank` refers to a Point object, which contains two attributes. Each attribute refers to a floating-point number.

We can read the value of an attribute using the same syntax:

```
>>> print(blank.y)
4.0
>>> x = blank.x
>>> print(x)
3.0
```

The expression `blank.x` means, “Go to the object `blank` refers to and get the value of `x`.” In this case, we assign that value to a variable named `x`. There is no conflict between the variable `x` and the attribute `x`.

You can use dot notation as part of any expression. For example:

```
>>> print('(%g, %g)' % (blank.x, blank.y))
(3.0, 4.0)
>>> distance = math.sqrt(blank.x**2 + blank.y**2)
>>> print(distance)
5.0
```

You can pass an instance as an argument in the usual way. For example:

```
def print_point(p):
    print('(%g, %g)' % (p.x, p.y))
```

`print_point` takes a point as an argument and displays it in mathematical notation. To invoke it, you can pass `blank` as an argument:

```
>>> print_point(blank)
(3.0, 4.0)
```

Inside the function, `p` is an alias for `blank`, so if the function modifies `p`, `blank` changes.

Exercise 13.1. *Write a function called* `distance` *that it takes two Points as arguments and returns the distance between them.*

13.3 Rectangles

Sometimes it is obvious what the attributes of an object should be, but other times you have to make decisions. For example, imagine you are designing a class to represent rectangles. What attributes would you use to specify the location and size of a rectangle? You can ignore angle; to keep things simple, assume that the rectangle is either vertical or horizontal.

There are at least two possibilities:

- You could specify one corner of the rectangle (or the center), the width, and the height.
- You could specify two opposing corners.

At this point it is hard to say whether either is better than the other, so we’ll implement the first one, just as an example.

Here is the class definition:

```
class Rectangle:
    """represent a rectangle.
       attributes: width, height, corner.
    """
```

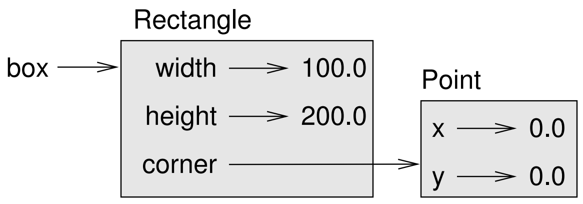
The docstring lists the attribute names. `width` and `height` are numbers; `corner` is a Point object that specifies the lower-left corner.

To represent a rectangle, you have to instantiate a Rectangle object and assign values to the attributes:

```
box = Rectangle()
box.width = 100.0
box.height = 200.0
box.corner = Point()
box.corner.x = 0.0
box.corner.y = 0.0
```

The expression `box.corner.x` means, "Go to the object `box` refers to and select the attribute named `corner`; then go to that object and select the attribute named `x`."

The figure shows the state of this object:



13.4 Instances as return values

Functions can return instances. For example, `find_center` takes a `Rectangle` as an argument and returns a `Point` that contains the coordinates of the center of the `Rectangle`:

```
def find_center(box):
    p = Point()
    p.x = box.corner.x + box.width/2.0
    p.y = box.corner.y + box.height/2.0
    return p
```

Here is an example that passes `box` as an argument and assign the resulting Point to `center`:

```
>>> center = find_center(box)
>>> print_point(center)
(50.0, 100.0)
```

13.5 Objects are mutable

We can change the state of an object by making an assignment to one of its attributes. For example, to change the size of a rectangle without changing its position, you can modify the values of `width` and `height`:

```
box.width = box.width + 50
box.height = box.width + 100
```

You can also write functions that modify objects. For example, `grow_rectangle` takes a Rectangle object and two numbers, `dwidth` and `dheight`, and adds the numbers to the width and height of the rectangle:

```
def grow_rectangle(rect, dwidth, dheight) :
    rect.width += dwidth
    rect.height += dheight
```

Here is an example that demonstrates the effect:

```
>>> print(box.width)
100.0
>>> print(box.height)
200.0
>>> grow_rectangle(box, 50, 100)
>>> print(box.width)
150.0
>>> print(box.height)
300.0
```

Inside the function, `rect` is an alias for `box`, so if the function modifies `rect`, `box` changes.

Exercise 13.2. *Write a function named* `move_rectangle` *that takes a Rectangle and two numbers named* `dx` *and* `dy`*. It should change the location of the rectangle by adding* `dx` *to the* `x` *coordinate of* `corner` *and adding* `dy` *to the* `y` *coordinate of* `corner`.

13.6 Copying

Aliasing can make a program difficult to read because changes made in one place might have unexpected effects in another place. It is hard to keep track of all the variables that might refer to a given object.

Copying an object is often an alternative to aliasing. The `copy` module contains a function called `copy` that can duplicate any object:

```
>>> p1 = Point()
>>> p1.x = 3.0
>>> p1.y = 4.0
>>> import copy
>>> p2 = copy.copy(p1)
```

`p1` and `p2` contain the same data, but they are not the same Point.

```
>>> print_point(p1)
(3.0, 4.0)
>>> print_point(p2)
(3.0, 4.0)
>>> p1 is p2
False
>>> p1 == p2
False
```

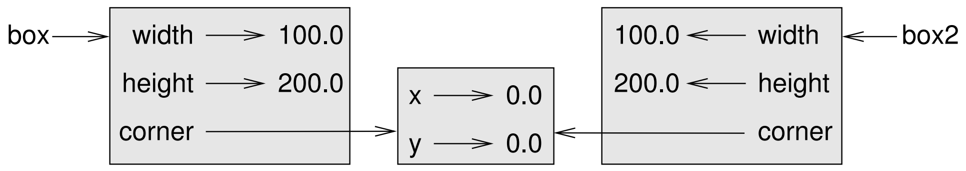
The `is` operator indicates that `p1` and `p2` are not the same object, which is what we expected. But you might have expected == to yield `True` because these points contain the same data. In that case, you will be disappointed to learn that for instances, the default behavior of the == operator is the same as the `is` operator; it checks object identity, not object equivalence.

This behavior can be changed, so for many objects defined in Python modules, the == operator checks equivalence (in whatever sense is appropriate). But the default is to check identity.

If you use `copy.copy` to duplicate a Rectangle, you will find that it copies the Rectangle object but not the embedded Point.

```
>>> box2 = copy.copy(box)
>>> box2 is box
False
>>> box2.corner is box.corner
True
```

Here is what the object diagram looks like:



This operation is called a **shallow copy** because it copies the object and any references it contains, but not the embedded objects.

For most applications, this is not what you want. In this example, invoking `grow_rectangle` on one of the Rectangles would not affect the other, but invoking `move_rectangle` on either would affect both! This behavior is confusing and error-prone.

Fortunately, the `copy` module contains a method named `deepcopy` that copies not only the object but also the objects it refers to, and the objects *they* refer to, and so on. You will not be surprised to learn that this operation is called a **deep copy**.

```
>>> box3 = copy.deepcopy(box)
>>> box3 is box
False
>>> box3.corner is box.corner
False
```

`box3` and `box` are completely separate objects.
Exercise 13.3. *Write a version* `move_rectangle` *that it creates and returns a new Rectangle instead of modifying the old one.*

13.7 Debugging

When you start working with objects, you are likely to encounter some new exceptions. If you try to access an attribute that doesn't exist, you get an `AttributeError`:

```
>>> p = Point(3, 4)
>>> print(p.z)
AttributeError: Point instance has no attribute 'z'
```

If you are not sure what class an object is, you can ask:

```
>>> type(p)
<class 'instance'>
```

This result tells us that `p` is an object, but not what kind. But all objects have a special attribute named `__class__` that refers to the object's particular class name.

```
>>> print(p.__class__)
__main__.Point
```

If you are not sure whether an object has a particular attribute, you can use the built-in function `hasattr`:

```
>>> hasattr(p, 'x')
True
>>> hasattr(p, 'z')
False
```

The first argument can be any object; the second argument is a *string* that contains the name of the attribute.

Another way to access the attributes of an object is through the special attribute `__dict__`, which is a dictionary that maps from attribute names (as strings) and values:

```
>>> print(p.__dict__)
{'y': 4, 'x': 3}
```

For purposes of debugging, you might find it useful to keep this function handy:

```
def print_attributes(obj):
    for attr in obj.__dict__:
        print(attr, getattr(obj, attr))
```

`print_attributes` traverses the items in the object's dictionary print each attrbute name and its corresponding value.

The built-in function `getattr` takes an object and an attribute name (as a string) and returns the attribute's value.

13.8 Glossary

class: A user-defined type. A class definition creates a new class object.

class object: An object that contains information about a user-defined time. The class object can be used to create instances of the type.

instance: An object that belongs to a class.

attribute: One of the named values associated with an object.

shallow copy: To copy the contents of an object, including any references to embedded objects; implemented by the `copy` function in the `copy` module.

deep copy: To copy the contents of an object as well as any embedded objects, and any objects embedded in them, and so on; implemented by the `deepcopy` function in the `copy` module.

object diagram: A diagram that shows objects, their attributes, and the values of the attributes.

13.9 Exercises

Chapter 14

Classes and functions

14.1 Time

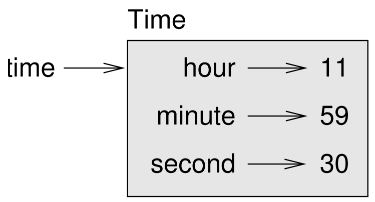
As another example of a user-defined type, we'll define a class called `Time` that records the time of day. The class definition looks like this:

```
class Time:
    """represents the time of day
       attributes: hour, minute, second"""
```

We can create a new `Time` object and assign attributes for hours, minutes, and seconds:

```
time = Time()
time.hour = 11
time.minute = 59
time.second = 30
```

The state diagram for the `Time` object looks like this:



Exercise 14.1. *Write a function* `print_time` *that takes a Time object and prints it in the form* `hour:minute:second`*.*
Exercise 14.2. *Write a boolean function* `after` *that takes two Time objects,* `t1` *and* `t2`*, and returns* `True` *if* `t1` *follows* `t2` *chronologically and* `False` *otherwise.*

14.2 Pure functions

In the next few sections, we'll write two versions of a function called `add_time`, which calculates the sum of two Time objects. They demonstrate two kinds of functions: pure functions and modifiers. They also demonstrate a development plan I'll call **prototype and patch**, which is a way of tackling a complex problem by starting with a simple prototype and incrementally dealing with the complications.

Here is a simple prototype of `add_time`:

```
def add_time(t1, t2):
    sum = Time()
    sum.hour = t1.hour + t2.hour
    sum.minute = t1.minute + t2.minute
    sum.second = t1.second + t2.second
    return sum
```

The function creates a new `Time` object, initializes its attributes, and returns a reference to the new object. This is called a **pure function** because it does not modify any of the objects passed to it as arguments and it has no side effects, such as displaying a value or getting user input.

To test this function, I'll create two Time objects: `start` contains the start time of a movie, like *Monty Python and the Holy Grail*, and `duration` contains the run time of the movie, which is one hour 35 minutes.

`add_time` figures out when the movie will be done.

```
>>> start = Time()
>>> start.hour = 9
>>> start.minute = 45
>>> start.second =  0

>>> duration = Time()
>>> duration.hour = 1
>>> duration.minute = 35
>>> duration.second = 0

>>> done = add_time(start, duration)
>>> print_time(done)
10:80:00
```

The result, `10:80:00` might not be what you were hoping for. The problem is that this function does not deal with cases where the number of seconds or minutes adds up to more than sixty. When that happens, we have to "carry" the extra seconds into the minute column or the extra minutes into the hour column.

Here's an improved version:

```
def add_time(t1, t2):
    sum = Time()
    sum.hour = t1.hour + t2.hour
    sum.minute = t1.minute + t2.minute
    sum.second = t1.second + t2.second

    if sum.second >= 60:
        sum.second -= 60
        sum.minute += 1

    if sum.minute >= 60:
        sum.minute -= 60
        sum.hour += 1

    return sum
```

Although this function is correct, it is starting to get big. We will see a shorter alternative later.

14.3 Modifiers

Sometimes it is useful for a function to modify the objects it gets as parameters. In that case, the changes are visible to the caller. Functions that work this way are called **modifiers**.

`increment`, which adds a given number of seconds to a `Time` object, can be written naturally as a modifier. Here is a rough draft:

```
def increment(time, seconds):
    time.second += seconds

    if time.second >= 60:
        time.second -= 60
        time.minute += 1

    if time.minute >= 60:
        time.minute -= 60
        time.hour += 1
```

The first line performs the basic operation; the remainder deals with the special cases we saw before.

Is this function correct? What happens if the parameter `seconds` is much greater than sixty? In that case, it is not enough to carry once; we have to keep doing it until `time.second` is less than sixty. One solution is to replace the `if` statements with `while` statements. That would make the function correct, but not very efficient.

Exercise 14.3. *Write a correct version of* `increment` *that doesn't contain any loops.*

Anything that can be done with modifiers can also be done with pure functions. In fact, some programming languages only allow pure functions. There is some evidence that programs that use pure functions are faster to develop and less error-prone than programs that use modifiers. But modifiers are convenient at times, and functional programs tend to be less efficient.

In general, I recommend that you write pure functions whenever it is reasonable and resort to modifiers only if there is a compelling advantage. This approach might be called a **functional programming style**.

Exercise 14.4. *Write a "pure" version of* `increment` *that creates and returns a new Time object rather than modifying the parameter.*

14.4 Prototyping versus planning

In this chapter, I demonstrated development plan called "prototype and patch." For each function, I wrote a rough draft that performed the basic calculation and then tested it, correcting flaws along the way.

This approach can be effective, especially if you don't yet have a deep understanding of the problem. But incremental patching can generate code that is unnecessarily complicated—since it deals with many special cases—and unreliable—since it is hard to know if you have found all the errors.

An alternative is **planned development**, in which high-level insight into the problem can make the programming much easier. In this case, the insight is that a Time object is really a three-digit number in base 60! The `second` attribute is the "ones column," the `minute` attribute is the "sixties column," and the `hour` attribute is the "thirty-six hundreds column."

When we wrote `add_time` and `increment`, we were effectively doing addition in base 60, which is why we had to carry from one column to the next.

This observation suggests another approach to the whole problem—we can convert Time objects to integers and take advantage of the fact that the computer knows how to do integer arithmetic.

Here is the function that converts Times to integers:

```
def time_to_int(time):
    minutes = time.hour * 60 + time.minute
    seconds = minutes * 60 + time.second
    return seconds
```

And here is the function that converts integers to Times (recall that `divmod` divides the first argument by the second and returns the quotient and remainder as a tuple).

```
def int_to_time(seconds):
    time = Time()
    minutes, time.second = divmod(seconds, 60)
    time.hour, time.minute = divmod(minutes, 60)
    return time
```

You might have to think a bit, and run some tests, to convince yourself that these functions are correct. But once they are debugged, you can use them to rewrite `add_time`:

```
def add_time(t1, t2):
    seconds = time_to_int(t1) + time_to_int(t2)
    return int_to_time(seconds)
```

This version is shorter than the original, and easier to verify.
Exercise 14.5. *Rewrite* `increment` *using* `time_to_int` *and* `int_to_time`.

In some ways, converting from base 60 to base 10 and back is harder than just dealing with times. Base conversion is more abstract; our intuition for dealing with times is better.

But if we have the insight to treat times as base 60 numbers and make the investment of writing the conversion functions (`time_to_int` and `int_to_time`), we get a program that is shorter, easier to read and debug, and more reliable.

It is also easier to add features later. For example, imagine subtracting two Times to find the duration between them. The naïve approach would be to implement subtraction with borrowing. Using the conversion functions would be easier and more likely to be correct.

Ironically, sometimes making a problem harder (or more general) makes it easier (because there are fewer special cases and fewer opportunities for error).

14.5 Glossary

prototype and patch: A development plan that involves writing a rough draft of a program, testing, and correcting errors as they are found.

planned development: A development plan that involves high-level insight into the problem and more planning than incremental development or prototype development.

pure function: A function that does not modify any of the objects it receives as arguments. Most pure functions are fruitful.

modifier: A function that changes one or more of the objects it receives as arguments. Most modifiers are fruitless.

functional programming style: A style of program design in which the majority of functions are pure.

14.6 Exercises

Exercise 14.6. *Write a function called* `mul_time` *that takes a Time object and a number and returns a new Time object that contains the product of the original Time and the number.*

Then use `mul_time` *to write a function that takes a Time object that represents the finishing time in a race, and a number that represents the distance, and returns a Time object that represents the average pace (time per mile).*

Chapter 15

Classes and methods

15.1 Object-oriented features

Python is an **object-oriented programming language**, which means that it provides features that support object-oriented programming.

It is not easy to define object-oriented programming, but we have already seen some of its characteristics:

- Programs are made up of object definitions and function definitions, and most of the computation is expressed in terms of operations on objects.
- Each object definition corresponds to some object or concept in the real world, and the functions that operate on that object correspond to the ways real-world objects interact.

For example, the `Time` class defined in Chapter 14 corresponds to the way people record the time of day, and the functions we defined correspond to the kinds of things people do with times. Similarly, the `Point` and `Rectangle` classes correspond to the mathematical concepts of a point and a rectangle.

So far, we have not taken advantage of the features Python provides to support object-oriented programming. Strictly speaking, these features are not necessary. For the most part, they provide an alternative syntax for things we have already done, but in many cases, the alternative is more concise and more accurately conveys the structure of the program.

For example, in the `Time` program, there is no obvious connection between the class definition and the function definitions that follow. With some examination, it is apparent that every function takes at least one `Time` object as an argument.

This observation is the motivation for **methods**; a method is a function that is associated with a particular class. For example, we have seen methods for strings, lists, dictionaries and tuples. In this chapter, we will define methods for user-defined types.

Methods are semantically the same as functions, but there are two syntactic differences:

- Methods are defined inside a class definition in order to make the relationship between the class and the method explicit.
- The syntax for invoking a method is different from the syntax for calling a function.

In the next few sections, we will take the functions from the previous two chapters and transform them into methods. This transformation is purely mechanical; you can do it simply by following a sequence of steps. If you are comfortable converting from one form to another, you will be able to choose the best form for whatever you are doing.

15.2 print_time

In Chapter 14, we defined a class named Time and in Exercise 14.1, you wrote a function named print_time:

```
class Time:
    """represents the time of day
       attributes: hour, minute, second"""

def print_time(time):
    print('%.2d:%.2d:%.2d' % (time.hour, time.minute, time.second))
```

To call this function, you have to pass a Time object as an argument:

```
>>> start = Time()
>>> start.hour = 9
>>> start.minute = 45
>>> start.second = 00
>>> print_time(start)
09:45:00
```

To make print_time a method, all we have to do is move the function definition inside the class definition. Notice the change in indentation.

```
class Time:
    def print_time(time):
        print('%.2d:%.2d:%.2d' % (time.hour, time.minute, time.second))
```

Now there are two ways to call print_time. The first (and less common) way is to use function syntax:

```
>>> Time.print_time(start)
09:45:00
```

In this use of dot notation, `Time` is the name of the class, and `print_time` is the name of the method. `start` is passed as a parameter.

The second (and more concise) way is to use method syntax:

```
>>> start.print_time()
09:45:00
```

In this use of dot notation, `print_time` is the name of the method (again), and `start` is the object the method is invoked on, which is called the **subject**. Just as the subject of a sentence is what the sentence is about, the subject of a method invocation is what the method is about.

Inside the method, the subject is assigned to the first parameter, so in this case `start` is assigned to `time`.

By convention, the first parameter of a method is called `self`, so it would be more common to write `print_time` like this:

```
class Time:
    def print_time(self):
        print('%.2d:%.2d:%.2d' % (self.hour, self.minute, self.second))
```

The reason for this convention is convoluted, but it is based on a useful metaphor:

The syntax for a function call, `print_time(start)`, suggests that the function is the active agent. It says something like, "Hey `print_time`! Here's an object for you to print."

In object-oriented programming, the objects are the active agents. A method invocation like `start.print_time()` says "Hey `start`! Please print yourself."

This change in perspective might be more polite, but it is not obvious that it is useful. In the examples we have seen so far, it may not be. But sometimes shifting responsibility from the functions onto the objects makes it possible to write more versatile functions, and makes it easier to maintain and reuse code.
Exercise 15.1. *Rewrite* `time_to_int` *(from Section 14.4) as a method. It is probably not appropriate to rewrite* `int_to_time` *as a method; it's not clear what object you would invoke it on!*

15.3 Another example

Here's a version of `increment` (from Section 14.3) rewritten as a method:

```
# inside class Time:

    def increment(self, seconds):
        seconds += self.time_to_int()
        return int_to_time(seconds)
```

This version assumes that `time_to_int` is written as a method, as in Exercise 15.1. Also, note that it is a pure function, not a modifier.

Here's how you would invoke `increment`:

```
>>> start.print_time()
09:45:00
>>> end = start.increment(1337)
>>> end.print_time()
10:07:17
```

The subject, `start`, gets assigned to the first parameter, `self`. The argument, `1337`, gets assigned to the second parameter, `seconds`.

This mechanism can be confusing, especially if you make an error. For example, if you invoke `increment` with two arguments, you get:

```
>>> end = start.increment(1337, 460)
TypeError: increment() takes exactly 2 arguments (3 given)
```

The error message is initially confusing, because there are only two arguments in parentheses. But the subject is also considered an argument, so all together that's three.

15.4 A more complicated example

`after` (from Exercise 14.2) is slightly more complicated because it takes two Time objects as parameters. In this case it is conventional to name the first parameter `self` and the second parameter `other`:

```
# inside class Time:

    def after(self, other):
        return self.time_to_int() > other.time_to_int()
```

To use this method, you have to invoke it on one object and pass the other as an argument:

```
>>> end.after(start)
True
```

One nice thing about this syntax is that it has the same word order as English, subject-verb-object.

15.5 The `init` method

The `init` method (short for "initialization") is a special method that gets invoked when an object is instantiated. Its full name is `__init__` (two underscore characters, followed by `init`, and then two more underscores). An init method for the `Time` class might look like this:

```
# inside class Time:

    def __init__(self, hour=0, minute=0, second=0):
        self.hour = hour
        self.minute = minute
        self.second = second
```

It is common for the parameters of `__init__` to have the same names as the attributes. The statement

```
        self.hour = hour
```

stores the value of the parameter `hour` as an attribute in the new Time object `self`.

The parameters are optional, so if you call `Time` with no arguments, you get the default values.

```
>>> time = Time()
>>> time.print_time()
00:00:00
```

If you provide one argument, it overrides `hour`:

```
>>> time = Time (9)
>>> time.print_time()
09:00:00
```

If you provide two arguments, they override `hour` and `minute`.

```
>>> time = Time(9, 45)
>>> time.print_time()
09:45:00
```

And if you provide three arguments, they override all three default values.
Exercise 15.2. *Write an init method for the* `Point` *class that takes* `x` *and* `y` *as optional parameters and assigns them to the corresponding attributes.*

15.6 The `str` method

`__str__` is a special method name, like `__init__`, that is supposed to return a string representation of an object.

For example, here is a `str` method for Time objects:

```
# inside class Time:

    def __str__(self):
        return '%.2d:%.2d:%.2d' % (self.hour, self.minute, self.second)
```

When you `print` an object, Python invokes the `str` method:

```
>>> time = Time(9, 45)
>>> print(time)
09:45:00
```

When I write a new class, I almost always start by writing `__init__`, which makes it easier to instantiate objects, and `__str__`, which is almost always useful for debugging.
Exercise 15.3. *Write a* `str` *method for the* `Point` *class. Create a Point object and print it.*

15.7 Operator overloading

By defining other special methods, you can specify the behavior of operators on user-defined types. For example, if you define an `add` method for the `Time` class, you can use the + operator on Time objects.

Here is what the definition might look like:

```
# inside class Time:

    def __add__(self, other):
        seconds = self.time_to_int() + other.time_to_int()
        return int_to_time(seconds)
```

And here is how you could use it:

```
>>> start = Time(9, 45)
>>> duration = Time(1, 35)
>>> print(start + duration)
11:20:00
```

When you apply the + operator to Time objects, Python invokes `__add__`. When you print the result, Python invokes `__str__`. So there is quite a lot happening behind the scenes!

Changing the behavior of an operator so that it works with user-defined types is called **operator overloading**. For every operator in Python there is a corresponding special method, like `__add__`.
Exercise 15.4. *Write an* `add` *method for the Point class.*

15.8 Type-based dispatch

In the previous section we added two Time objects, but you also might want to add an integer to a Time object. The following is an alternative version of `__add__` that checks the type of `other` and invokes either `add_time` or `increment`:

```
# inside class Time:

    def __add__(self, other):
        if isinstance(other, Time):
            return self.add_time(other)
        else:
            return self.increment(other)

    def add_time(self, other):
        seconds = self.time_to_int() + other.time_to_int()
        return int_to_time(seconds)

    def increment(self, seconds):
        seconds += self.time_to_int()
        return int_to_time(seconds)
```

The built-in function `isinstance` takes a value and a class object, and returns `True` if the value is an instance of the class.

If `other` is a Time object, `__add__` invokes `add_time`. Otherwise it assumes that the seconds parameter is a number and invokes `increment`. This operation is called a **type-based dispatch** because it dispatches the computation to different methods based on the type of the arguments.

Here are examples that use the + operator with different types:

```
>>> start = Time(9, 45)
>>> duration = Time(1, 35)
>>> print(start + duration)
11:20:00
>>> print(start + 1337)
10:07:17
```

Unfortunately, this implementation of addition is not commutative. If the integer is the first operand, you get

```
>>> print(1337 + start)
TypeError: unsupported operand type(s) for +: 'int' and 'instance'
```

The problem is, instead of asking the Time object to add an integer, Python is asking an integer to add a Time object, and it doesn't know how to do that. But there is a clever solution for this problem, the `radd` method, which stands for "right-side add."

This method is invoked when a Time object appears on the right side of the + operator. Here's the definition:

```
# inside class Time:

    def __radd__(self, other):
        return self.__add__(other)
```

And here's how it's used:

```
>>> print(1337 + start)
10:07:17
```

Exercise 15.5. *Write an* `add` *method for Points that works with either a Point object or a tuple:*

- *If the second operand is a Point, the method should return a new Point whose x coordinate is the sum of the x coordinates of the operands, and likewise for the y coordinates.*
- *If the second operand is a tuple, the method should add the first element of the tuple to the x coordinate and the second element to the y coordinate, and return a new Point with the result.*

15.9 Polymorphism

Type-based dispatch is useful when it is necessary, but (fortunately) it is not always necessary. Often you can avoid it by writing functions that work correctly for arguments with different types.

Many of the functions we wrote for strings will actually work for any kind of sequence. For example, in Section 12.1 we used `histogram` to count the number of times each letter appears in a word.

```
def histogram(s):
    d = {}
    for c in s:
        if c not in d:
            d[c] = 1
        else:
            d[c] = d[c]+1
    return d
```

This function also works for lists, tuples, and even dictionaries, as long as the elements of `s` are hashable, so they can be used as keys in `d`.

```
>>> t = ['spam', 'egg', 'spam', 'spam', 'bacon', 'spam']
>>> histogram(t)
{'bacon': 1, 'egg': 1, 'spam': 4}
```

Functions that can work with several types are called **polymorphic**.

Many of the built-in functions are polymorphic. For example, `sum` works with any kind of sequence, as long as the elements support the addition operator.

```
>>> t = [1, 2.0, 42L]
>>> print(sum(t))
45.0
```

Since Time objects provide an `add` method, they work with `sum`:

```
>>> t1 = Time(7, 43)
>>> t2 = Time(7, 41)
>>> t3 = Time(7, 37)
>>> total = sum([t1, t2, t3])
>>> print(total)
23:01:00
```

In general, if all of the operations inside a function work with a given type, then the function works with that type.

The best kind of polymorphism is the unintentional kind, where you discover that a function you have already written can be applied to a type you never planned for.

15.10 Exercises

Exercise 15.6. *Write a definition for a class named* `Kangaroo` *with the following methods:*

1. *An* `__init__` *method that initializes an attribute named* `pouch_contents` *to an empty list.*
2. *A method named* `put_in_pouch` *that takes an object of any type and adds it to* `pouch_contents`*.*

Test your code by creating two `Kangaroo` *objects, assigning them to variables named* `kanga` *and* `roo`*, and then adding* `roo` *to the contents of* `kanga`*'s pouch.*

15.11 Glossary

object-oriented language: A language that provides features, such as user-defined classes and inheritance, that facilitate object-oriented programming.

object-oriented programming: A style of programming in which data and the operations that manipulate it are organized into classes and methods.

method: A function that is defined inside a class definition and is invoked on instances of that class.

subject: The object a method is invoked on.

operator overloading: Changing the behavior of an operator like + so it works with a user-defined type.

type-based dispatch: A programming pattern that checks the type of an operand and invokes different functions for different types.

polymorphic: Pertaining to a function that can work with more than one type.

Chapter 16

Inheritance

In this chapter we will develop classes to represent playing cards, decks of cards, and poker hands. If you don't play poker, don't worry; I'll tell you what you need to know for the exercises.

But if you are not familiar with common playing cards, now would be a good time to get a deck, or else this chapter might not make much sense.

16.1 Card objects

There are fifty-two cards in a deck, each of which belongs to one of four suits and one of thirteen ranks. The suits are Spades, Hearts, Diamonds, and Clubs (in descending order in bridge). The ranks are Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Depending on the game that you are playing, an Ace may be higher than King or lower than 2.

If we want to define a new object to represent a playing card, it is obvious what the attributes should be: `rank` and `suit`. It is not as obvious what type the attributes should be. One possibility is to use strings containing words like `"Spade"` for suits and `"Queen"` for ranks. One problem with this implementation is that it would not be easy to compare cards to see which had a higher rank or suit.

An alternative is to use integers to **encode** the ranks and suits. In this context, "encode" means that we are going to define a mapping between numbers and suits, or between numbers and ranks. This kind of encoding is not meant to be a secret (that would be "encryption").

For example, this table shows the suits and the corresponding integer codes:

Spades	$\mapsto$	3
Hearts	$\mapsto$	2
Diamonds	$\mapsto$	1
Clubs	$\mapsto$	0

This code makes it easy to compare cards; because higher suits map to higher numbers, we can compare suits by comparing their codes.

The mapping for ranks is fairly obvious; each of the numerical ranks maps to the corresponding integer, and for face cards:

Jack	$\mapsto$	11
Queen	$\mapsto$	12
King	$\mapsto$	13

I am using the $\mapsto$ symbol to make is clear that these mappings are not part of the Python program. They are part of the program design, but they don't appear explicitly in the code.

The class definition for `Card` looks like this:

```
class Card:
    """represents a standard playing card."""

    def __init__(self, suit=0, rank=2):
        self.suit = suit
        self.rank = rank
```

As usual, the `init` method takes an optional parameter for each attribute. The default card is the 2 of Clubs.

To create a Card, you call `Card` with the suit and rank of the card you want.

```
threeOfClubs = Card(3, 1)
```

In the next section we'll figure out which card that is.

16.2 Class attributes

In order to print Card objects in a way that people can easily read, we need a mapping from the integer codes to the corresponding ranks and suits. A natural way to do that is with lists of strings. We assign these lists to **class attributes**:

```
# inside class Card:

    suit_names = ['Clubs', 'Diamonds', 'Hearts', 'Spades']
    rank_names = [None, 'Ace', '2', '3', '4', '5', '6', '7',
              '8', '9', '10', 'Jack', 'Queen', 'King']

    def __str__(self):
        return '%s of %s' % (Card.rank_names[self.rank],
                             Card.suit_names[self.suit])
```

Because `suit_names` and `rank_names` are defined outside of any method, they are class attributes; that is, they are associated with the class `Card` rather than with a particular Card instance.

Attributes like `suit` and `rank` are more precisely called **instance attributes** because they are associated with a particular instance.

Both kinds of attribute are accessed using dot notation. For example, in `__str__`, `self` is a Card object, and `self.rank` is its rank. Similarly, `Card` is a class object, and `Card.rank_names` is a list of strings associated with the class.

Every card has its own `suit` and `rank`, but there is only one copy of `suit_names` and `rank_names`.

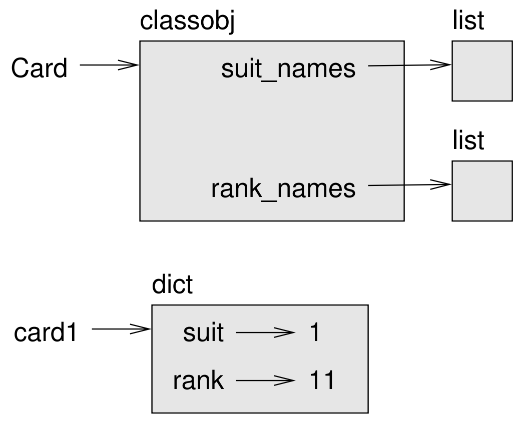
Finally, the expression `Card.rank_names[self.rank]` means "use the attribute `rank` from the object `self` as an index into the list `rank_names` from the class `Card`, and select the appropriate string."

The first element of `rank_names` is `None` because there is no card with rank zero. By including `None` as a place-keeper, we get a mapping with the nice property that the index 2 maps to the string `'2'`, and so on.

With the methods we have so far, we can create and print cards:

```
>>> card1 = Card(1, 11)
>>> print(card1)
Jack of Diamonds
```

Here is a diagram that shows the `Card` class object and one Card instance:



Card is a class object, so it has type classobj. card1 has type Card. (To save space, I didn't draw the contents of suit_names and rank_names).

16.3 Comparing cards

For built-in types, there are conditional operators (<, >, ==, etc.) that compare values and determine when one is greater than, less than, or equal to another. For user-defined types, we can override the behavior of the built-in operators by providing a method named __cmp__.

The cmp method takes two parameters, self and other, and returns a positive number if the first object is greater, a negative number if the second object is greater, and 0 if they are equal to each other.

The correct ordering for cards is not obvious. For example, which is better, the 3 of Clubs or the 2 of Diamonds? One has a higher rank, but the other has a higher suit. In order to compare cards, you have to decide whether rank or suit is more important.

The answer might depend on what game you are playing, but to keep things simple, we'll make the arbitrary choice that suit is more important, so all of the Spades outrank all of the Diamonds, and so on.

With that decided, we can write __cmp__:

```
# inside class Card:

    def __cmp__(self, other):
        # check the suits
        if self.suit > other.suit: return 1
        if self.suit < other.suit: return -1

        # suits are the same... check ranks
        if self.rank > other.rank: return 1
        if self.rank < other.rank: return -1

        # ranks are the same... it's a tie
        return 0
```

You can write this more concisely using tuple comparison:

```
# inside class Card:

    def __cmp__(self, other):
        t1 = self.suit, self.rank
        t2 = other.suit, other.rank
        return cmp(t1, t2)
```

The built-in function `cmp` has the same interface as the method `__cmp__`: it takes two values and returns a positive number if the first is larger, a negative number of the second is larger, and 0 if they are equal.

Exercise 16.1. *Write a `__cmp__` method for Time objects. Hint: you can use tuple comparison, but you also might consider using integer subtraction.*

16.4 Decks

Now that we have Card objects, the next step is to define a class to represent decks. Since a deck is made up of cards, a natural choice is for each Deck object to contain a list of cards as an attribute.

The following is a class definition for `Deck`. The `init` method creates the attribute `cards` and generates the standard set of fifty-two cards:

```
class Deck:

    def __init__(self):
        self.cards = []
        for suit in range(4):
            for rank in range(1, 14):
                card = Card(suit, rank)
                self.cards.append(card)
```

The easiest way to populate the deck is with a nested loop. The outer loop enumerates the suits from 0 to 3. The inner loop enumerates the ranks from 1 to 13. Each iteration of the inner loop creates a new Card with the current suit and rank, and appends it to `self.cards`.

16.5 Printing the deck

Here is a `str` method for `Deck`:

```
#inside class Deck:

    def __str__(self):
        res = []
        for card in self.cards:
            res.append(str(card))
        return '\n'.join(res)
```

This method demonstrates an efficient way to accumulate a large string, by building a list of strings and then using `join`. The built-in function `str` invokes the `__str__` method on each card and returns the string representation.

Since we invoke `join` on a newline character, the cards are separated by newlines. Here's what the result looks like:

```
>>> deck = Deck()
>>> print(deck)
Ace of Clubs
2 of Clubs
3 of Clubs
...
10 of Spades
Jack of Spades
Queen of Spades
King of Spades
```

Even though the result appears on 52 lines, it is one long string that contains newlines.

16.6 Add, remove, shuffle and sort

To deal cards, we would like a method that removes a card from the deck and returns it. The list method `pop` provides a convenient way to do that:

```
#inside class Deck:

    def pop_card(self):
        return self.cards.pop()
```

Since `pop` removes the *last* card in the list, we are in effect dealing from the bottom of the deck.

To add a card, we can use the list method `append`:

```
#inside class Deck:

    def add_card(self, card):
        self.cards.append(card)
```

A method like this that uses another function without doing much real work is sometimes called a **veneer**. The metaphor comes from woodworking, where it is common to glue a thin layer of good quality wood to the surface of a cheaper piece of wood.

In this case we are defining a "thin" method that expresses a list operation in terms that are appropriate for decks.

As another example, we can write a Deck method named `shuffle` using the function `shuffle` from the `random` module:

```
# inside class Deck:

    def shuffle(self):
        random.shuffle(self.cards)
```

Don't forget to import `random`.

Exercise 16.2. *Write a Deck method named* `sort` *that uses the list method* `sort` *to sort the cards in a* `Deck`. `sort` *uses the* `__cmp__` *method we defined to determine sort order.*

16.7 Inheritance

The language feature most often associated with object-oriented programming is **inheritance**. Inheritance is the ability to define a new class that is a modified version of an existing class.

It is called "inheritance" because the new class inherits the methods of the existing class. Extending this metaphor, the existing class is called the **parent** class and the new class is called the **child**.

As an example, let's say we want a class to represent a "hand," that is, the set of cards held by one player. A hand is similar to a deck: both are made up of a set of cards, and both require operations like adding and removing cards.

A hand is also different from a deck; there are operations we want for hands that don't make sense for a deck. For example, in poker we might compare two hands to see which one wins. In bridge, we might compute a score for a hand in order to make a bid.

This relationship between classes—similar, but different—lends itself to inheritance.

The definition of a child class is like other class definitions, but the name of the parent class appears in parentheses:

```
class Hand(Deck):
    """represents a hand of playing cards"""
```

This definition indicates that `Hand` inherits from `Deck`; that means we can use methods like `pop_card` and `add_card` for Hands as well as Decks.

`Hand` also inherits the `init` method from `Deck`, but it doesn't really do what we want: instead of populating the hand with 52 new cards, the `init` method for Hands should initialize `cards` with an empty list.

If we provide an `init` method in the `Hand` class, it overrides the one in the `Deck` class:

```
# inside class Hand:

    def __init__(self, label=''):
        self.cards = []
        self.label = label
```

So when you create a Hand, Python invokes this `init` method:

```
>>> hand = Hand('new hand')
>>> print(hand.cards)
[]
>>> print(hand.label)
new hand
```

But the other methods are inherited from `Deck`, so we can use `pop_card` and `add_card` to deal a card:

```
>>> deck = Deck()
>>> card = deck.pop_card()
>>> hand.add_card(card)
>>> print(hand)
King of Spades
```

The next natural step is to encapsulate this code in a method called `move_cards`:

```
#inside class Deck:

    def move_cards(self, hand, num):
        for i in range(num):
            hand.add_card(self.pop_card())
```

`move_cards` takes two arguments, a Hand object and the number of cards to deal. It modifies both `self` and `hand`, and returns `None`.

In some games, cards are moved from one hand to another, or from a hand back to the deck. You can use `move_cards` for any of these operations: `self` can be either a Deck or a Hand, and `hand`, despite the name, can also be a `Deck`.
Exercise 16.3. *Write a Deck method called* `deal_hands` *that takes two parameters, the number of hands and the number of cards per hand, and that creates new Hand objects, deals the appropriate number of cards per hand, and returns a list of Hand objects.*

Inheritance is a useful feature. Some programs that would be repetitive without inheritance can be written more elegantly with it. Inheritance can facilitate code reuse, since you can customize the behavior of parent classes without having to modify them. In some cases, the inheritance structure reflects the natural structure of the problem, which makes the program easier to understand.

On the other hand, inheritance can make programs difficult to read. When a method is invoked, it is sometimes not clear where to find its definition. The relevant code may be scattered among several modules. Also, many of the things that can be done using inheritance can be done as well or better without it.

16.8 Class diagrams

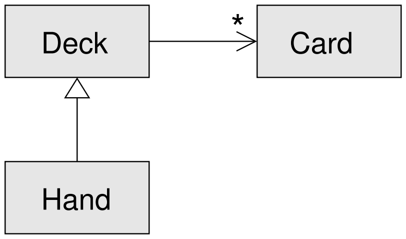
So far we have seen stack diagrams, which show the state of a program, and object diagrams, which show the attributes of an object and their values. These diagrams represent a snapshot in the execution of a program, so they change as the program runs.

They are also highly detailed, and for some applications, too detailed. A class diagrams is a more abstract representation of the structure of a program. Instead of showing individual objects, it shows classes and the relationships between them.

There are several kinds of relationship between classes:

- Objects in one class might contain references to objects in another class. For example, each Rectangle contains a reference to a Point, and each Deck contains references to many Cards. This kind of relationship is called **HAS-A**, as in, “a Rectangle has a Point.”
- One class might inherit from another. This relationship is called **IS-A**, as in, “a Hand is a kind of a Deck.”
- Once class might depend on another in the sense that changes in one class would require changes in the other.

A **class diagram** is a graphical representation of these relationships between classes. For example, this diagram shows the relationships between `Card`, `Deck` and `Hand`.



The arrow with a hollow triangle head represents an IS-A relationship; in this case it indicates that Hand inherits from Deck.

The standard arrow head represents a HAS-A relationshop; in this case a Deck has references to Card objects.

The star (`*`) near the arrow head is a **multiplicity**; it indicates how many Cards a Deck has. A multiplicity can be a simple number, like `52`, a range, like `5..7` or a star, which indicates that a Deck can have any number of Cards.

16.9 Glossary

encode: To represent one set of values using another set of values by constructing a mapping between them.

class attribute: An attribute associated with a class object. Class attributes are defined inside a class definition but outside any method.

instance attribute: An attribute associated with an instance of a class.

veneer: A method or function that provides a different interface to another function without doing much computation.

inheritance: The ability to define a new class that is a modified version of a previously defined class.

parent class: The class from which a child class inherits.

child class: A new class created by inheriting from an existing class; also called a "subclass."

IS-A relationship: The relationship between a child class and its parent class.

HAS-A relationship: The relationship between two classes where instances of one class contain references to instances of the other.

class diagram: A diagram that shows the classes in a program and the relationships between them.

multiplicity: A notation in a class diagram that shows, for a HAS-A relationship, how many references there are to instances of another class.

16.10 Exercises

The following are the possible hands in poker, in increasing order of value (and decreasing order of probability):

pair: two cards with the same rank

two pair: two pairs of cards with the same rank

three of a kind: three cards with the same rank

straight: five cards with ranks in sequence (aces can be high or low, so `Ace-2-3-4-5` is a straight and so is `10-Jack-Queen-King-Ace`, but `Queen-King-Ace-2-3` is not.)

flush: five cards with the same suit

full house: three cards with one rank, two cards with another

four of a kind: four cards with the same rank

straight flush: five cards in sequence (as defined above) and with the same suit

The goal of these exercises is to estimate the probability of drawing these various hands.

1. Download the following files from `thinkpython.com/code`:

   `Card.py` : A complete version of the `Card`, `Deck` and `Hand` classes in this chapter.

   `PokerHand.py` : An incomplete implementation of a class that represents a poker hand, and some code that tests it.

2. If you run `PokerHand.py`, it deals six 7-card poker hands and checks to see if any of them contains a flush. Read this code carefully before you go on.

3. Add methods to `PokerHand.py` named `has_pair`, `has_twopair`, etc. that return True or False according to whether or not the hand meets the relevant criteria. Your code should work correctly for "hands" that contain any number of cards (although 5 and 7 are the most common sizes).

4. Write a method named `classify` that figures out the highest-value classification for a hand and sets the `label` attribute accordingly. For example, a 7-card hand might contain a flush and a pair; it should be labeled "flush".

5. When you are convinced that your classification methods are working, the next step is to estimate the probablities of the various hands. Write a function in `PokerHand.py` that shuffles a deck of cards, divides it into hands, classifies the hands, and counts the number of times various classifications appear.

6. Print a table of the classifications and their probabilities. Run your program with larger and larger numbers of hands until the output values converge to a reasonable degree of accuracy.

Part IV

Appendies

Appendix A

Debugging

Different kinds of errors can occur in a program, and it is useful to distinguish among them in order to track them down more quickly:

- Syntax errors are produced by Python when it is translating the source code into byte code. They usually indicate that there is something wrong with the syntax of the program. Example: Omitting the colon at the end of a `def` statement yields the somewhat redundant message `SyntaxError: invalid syntax`.
- Runtime errors are produced by the runtime system if something goes wrong while the program is running. Most runtime error messages include information about where the error occurred and what functions were executing. Example: An infinite recursion eventually causes a runtime error of "maximum recursion depth exceeded."
- Semantic errors are problems with a program that compiles and runs but doesn't do the right thing. Example: An expression may not be evaluated in the order you expect, yielding an unexpected result.

The first step in debugging is to figure out which kind of error you are dealing with. Although the following sections are organized by error type, some techniques are applicable in more than one situation.

A.1 Syntax errors

Syntax errors are usually easy to fix once you figure out what they are. Unfortunately, the error messages are often not helpful. The most common messages are `SyntaxError: invalid syntax` and `SyntaxError: invalid token`, neither of which is very informative.

On the other hand, the message does tell you where in the program the problem occurred. Actually, it tells you where Python noticed a problem, which is not necessarily where the error is. Sometimes the error is prior to the location of the error message, often on the preceding line.

If you are building the program incrementally, you should have a good idea about where the error is. It will be in the last line you added.

If you are copying code from a book, start by comparing your code to the book's code very carefully. Check every character. At the same time, remember that the book might be wrong, so if you see something that looks like a syntax error, it might be.

Here are some ways to avoid the most common syntax errors:

1. Make sure you are not using a Python keyword for a variable name.
2. Check that you have a colon at the end of the header of every compound statement, including `for`, `while`, `if`, and `def` statements.
3. Check that indentation is consistent. You may indent with either spaces or tabs but it's best not to mix them. Each level should be nested the same amount.
4. Make sure that any strings in the code have matching quotation marks.
5. If you have multiline strings with triple quotes (single or double), make sure you have terminated the string properly. An unterminated string may cause an `invalid token` error at the end of your program, or it may treat the following part of the program as a string until it comes to the next string. In the second case, it might not produce an error message at all!
6. An unclosed bracket—`(`, `{`, or `[`—makes Python continue with the next line as part of the current statement. Generally, an error occurs almost immediately in the next line.
7. Check for the classic = instead of == inside a conditional.

If nothing works, move on to the next section...

A.1.1 I can't get my program to run no matter what I do.

If the compiler says there is an error and you don't see it, that might be because you and the compiler are not looking at the same code. Check your programming environment to make sure that the program you are editing is the one Python is trying to run. If you are not sure, try putting an obvious and deliberate syntax error at the beginning of the program. Now run (or import) it again. If the compiler doesn't find the new error, there is probably something wrong with the way your environment is set up.

If this happens, one approach is to start again with a new program like "Hello, World!," and make sure you can get a known program to run. Then gradually add the pieces of the new program to the working one.

A.2 Runtime errors

Once your program is syntactically correct, Python can import it and at least start running it. What could possibly go wrong?

A.2.1 My program does absolutely nothing.

This problem is most common when your file consists of functions and classes but does not actually invoke anything to start execution. This may be intentional if you only plan to import this module to supply classes and functions.

If it is not intentional, make sure that you are invoking a function to start execution, or execute one from the interactive prompt. Also see the "Flow of Execution" section below.

A.2.2 My program hangs.

If a program stops and seems to be doing nothing, we say it is "hanging." Often that means that it is caught in an infinite loop or an infinite recursion.

- If there is a particular loop that you suspect is the problem, add a `print` statement immediately before the loop that says "entering the loop" and another immediately after that says "exiting the loop."

  Run the program. If you get the first message and not the second, you've got an infinite loop. Go to the "Infinite Loop" section below.

- Most of the time, an infinite recursion will cause the program to run for a while and then produce a "RuntimeError: Maximum recursion depth exceeded" error. If that happens, go to the "Infinite Recursion" section below.

  If you are not getting this error but you suspect there is a problem with a recursive method or function, you can still use the techniques in the "Infinite Recursion" section.

- If neither of those steps works, start testing other loops and other recursive functions and methods.

- If that doesn't work, then it is possible that you don't understand the flow of execution in your program. Go to the "Flow of Execution" section below.

Infinite Loop

If you think you have an infinite loop and you think you know what loop is causing the problem, add a `print` statement at the end of the loop that prints the values of the variables in the condition and the value of the condition.

For example:

```
while x > 0 and y < 0 :
    # do something to x
    # do something to y

    print("x: ", x)
    print("y: ", y)
    print("condition: ", (x > 0 and y < 0))
```

Now when you run the program, you will see three lines of output for each time through the loop. The last time through the loop, the condition should be `false`. If the loop keeps going, you will be able to see the values of `x` and `y`, and you might figure out why they are not being updated correctly.

Infinite Recursion

Most of the time, an infinite recursion will cause the program to run for a while and then produce a `Maximum recursion depth exceeded` error.

If you suspect that a function or method is causing an infinite recursion, start by checking to make sure that there is a base case. In other words, there should be some condition that will cause the function or method to return without making a recursive invocation. If not, then you need to rethink the algorithm and identify a base case.

If there is a base case but the program doesn't seem to be reaching it, add a `print` statement at the beginning of the function or method that prints the parameters. Now when you run the program, you will see a few lines of output every time the function or method is invoked, and you will see the parameters. If the parameters are not moving toward the base case, you will get some ideas about why not.

Flow of Execution

If you are not sure how the flow of execution is moving through your program, add `print` statements to the beginning of each function with a message like "entering function `foo`," where `foo` is the name of the function.

Now when you run the program, it will print a trace of each function as it is invoked.

A.2.3 When I run the program I get an exception.

If something goes wrong during runtime, Python prints a message that includes the name of the exception, the line of the program where the problem occurred, and a traceback.

The traceback identifies the function that is currently running, and then the function that invoked it, and then the function that invoked *that*, and so on. In other words, it traces the path of function invocations that got you to where you are. It also includes the line number in your file where each of these calls occurs.

The first step is to examine the place in the program where the error occurred and see if you can figure out what happened. These are some of the most common runtime errors:

NameError: You are trying to use a variable that doesn't exist in the current environment. Remember that local variables are local. You cannot refer to them from outside the function where they are defined.

TypeError: There are several possible causes:

- You are trying to use a value improperly. Example: indexing a string, list, or tuple with something other than an integer.
- There is a mismatch between the items in a format string and the items passed for conversion. This can happen if either the number of items does not match or an invalid conversion is called for.
- You are passing the wrong number of arguments to a function or method. For methods, look at the method definition and check that the first parameter is `self`. Then look at the method invocation; make sure you are invoking the method on an object with the right type and providing the other arguments correctly.

KeyError: You are trying to access an element of a dictionary using a key value that the dictionary does not contain.

AttributeError: You are trying to access an attribute or method that does not exist.

IndexError: The index you are using to access a list, string, or tuple is greater than its length minus one. Immediately before the site of the error, add a `print` statement to display the value of the index and the length of the array. Is the array the right size? Is the index the right value?

A.2.4 I added so many `print` statements I get inundated with output.

One of the problems with using `print` statements for debugging is that you can end up buried in output. There are two ways to proceed: simplify the output or simplify the program.

To simplify the output, you can remove or comment out `print` statements that aren't helping, or combine them, or format the output so it is easier to understand.

To simplify the program, there are several things you can do. First, scale down the problem the program is working on. For example, if you are sorting an array, sort a *small* array. If the program takes input from the user, give it the simplest input that causes the problem.

Second, clean up the program. Remove dead code and reorganize the program to make it as easy to read as possible. For example, if you suspect that the problem is in a deeply nested part of the program, try rewriting that part with simpler structure. If you suspect a large function, try splitting it into smaller functions and testing them separately.

Often the process of finding the minimal test case leads you to the bug. If you find that a program works in one situation but not in another, that gives you a clue about what is going on.

Similarly, rewriting a piece of code can help you find subtle bugs. If you make a change that you think doesn't affect the program, and it does, that can tip you off.

A.3 Semantic errors

In some ways, semantic errors are the hardest to debug, because the compiler and the runtime system provide no information about what is wrong. Only you know what the program is supposed to do, and only you know that it isn't doing it.

The first step is to make a connection between the program text and the behavior you are seeing. You need a hypothesis about what the program is actually doing. One of the things that makes that hard is that computers run so fast.

You will often wish that you could slow the program down to human speed, and with some debuggers you can. But the time it takes to insert a few well-placed `print` statements is often short compared to setting up the debugger, inserting and removing breakpoints, and "walking" the program to where the error is occurring.

A.3.1 My program doesn't work.

You should ask yourself these questions:

- Is there something the program was supposed to do but which doesn't seem to be happening? Find the section of the code that performs that function and make sure it is executing when you think it should.
- Is something happening that shouldn't? Find code in your program that performs that function and see if it is executing when it shouldn't.
- Is a section of code producing an effect that is not what you expected? Make sure that you understand the code in question, especially if it involves invocations to functions or methods in other Python modules. Read the documentation for the functions you invoke. Try them out by writing simple test cases and checking the results.

In order to program, you need to have a mental model of how programs work. If you write a program that doesn't do what you expect, very often the problem is not in the program; it's in your mental model.

The best way to correct your mental model is to break the program into its components (usually the functions and methods) and test each component independently. Once you find the discrepancy between your model and reality, you can solve the problem.

Of course, you should be building and testing components as you develop the program. If you encounter a problem, there should be only a small amount of new code that is not known to be correct.

A.3.2 I've got a big hairy expression and it doesn't do what I expect.

Writing complex expressions is fine as long as they are readable, but they can be hard to debug. It is often a good idea to break a complex expression into a series of assignments to temporary variables.

For example:

```
self.hands[i].addCard(self.hands[self.findNeighbor(i)].popCard())
```

This can be rewritten as:

```
neighbor = self.findNeighbor(i)
pickedCard = self.hands[neighbor].popCard()
self.hands[i].addCard(pickedCard)
```

The explicit version is easier to read because the variable names provide additional documentation, and it is easier to debug because you can check the types of the intermediate variables and display their values.

Another problem that can occur with big expressions is that the order of evaluation may not be what you expect. For example, if you are translating the expression $\frac{x}{2\pi}$ into Python, you might write:

```
y = x / 2 * math.pi
```

That is not correct because multiplication and division have the same precedence and are evaluated from left to right. So this expression computes $x\pi/2$.

A good way to debug expressions is to add parentheses to make the order of evaluation explicit:

```
 y = x / (2 * math.pi)
```

Whenever you are not sure of the order of evaluation, use parentheses. Not only will the program be correct (in the sense of doing what you intended), it will also be more readable for other people who haven't memorized the rules of precedence.

A.3.3 I've got a function or method that doesn't return what I expect.

If you have a `return` statement with a complex expression, you don't have a chance to print the `return` value before returning. Again, you can use a temporary variable. For example, instead of:

```
return self.hands[i].removeMatches()
```

you could write:

```
count = self.hands[i].removeMatches()
return count
```

Now you have the opportunity to display the value of `count` before returning.

A.3.4 I'm really, really stuck and I need help.

First, try getting away from the computer for a few minutes. Computers emit waves that affect the brain, causing these effects:

- Frustration and/or rage.
- Superstitious beliefs ("the computer hates me") and magical thinking ("the program only works when I wear my hat backward").
- Random-walk programming (the attempt to program by writing every possible program and choosing the one that does the right thing).

If you find yourself suffering from any of these symptoms, get up and go for a walk. When you are calm, think about the program. What is it doing? What are some possible causes of that behavior? When was the last time you had a working program, and what did you do next?

Sometimes it just takes time to find a bug. We often find bugs when we are away from the computer and let our minds wander. Some of the best places to find bugs are trains, showers, and in bed, just before you fall asleep.

A.3.5 No, I really need help.

It happens. Even the best programmers occasionally get stuck. Sometimes you work on a program so long that you can't see the error. A fresh pair of eyes is just the thing.

Before you bring someone else in, make sure you have exhausted the techniques described here. Your program should be as simple as possible, and you should be working on the smallest input that causes the error. You should have `print` statements in the appropriate places (and the output they produce should be comprehensible). You should understand the problem well enough to describe it concisely.

When you bring someone in to help, be sure to give them the information they need:

- If there is an error message, what is it and what part of the program does it indicate?
- What was the last thing you did before this error occurred? What were the last lines of code that you wrote, or what is the new test case that fails?
- What have you tried so far, and what have you learned?

When you find the bug, take a second to think about what you could have done to find it faster. Next time you see something similar, you will be able to find the bug more quickly.

Remember, the goal is not just to make the program work. The goal is to learn how to make the program work.