Math 105: Finite Mathematics

Study Guide 3

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1 Natural Deduction

And Intro	**And Elim**	**Or Intro**	**Resolution**	**MP**	**MT**
P			$P \vee Q$	$P \Rightarrow Q$	$P \Rightarrow Q$
Q	$P \wedge Q$	P	$\neg P$	P	$\neg Q$
$P \wedge Q$	P	$P \vee Q$	Q	Q	$\neg P$

$$
\begin{aligned}
&\neg\neg P = P\\
&P \Rightarrow Q = \neg P \vee Q\\
&\neg(P \vee Q) = \neg P \wedge \neg Q\\
&\neg(P \wedge Q) = \neg P \vee \neg Q\\
&P \wedge Q = Q \wedge P\\
&P \vee Q = Q \vee P
\end{aligned}
$$

All the rules in the tables above can be proven through the use of a truth table.

When trying to prove a statement, it is sometimes helpful to start backwards. Start with your expected result, and see what you need to prove this, and repeat until you're only left with facts you were given to be true.

2 Saddle Points

We finish the semester by examining games where players act simultaneously. You still want to win, or maximize your outcome, but you don't know what the other player will do. We will deal with two player games and two choices per player, which are zero-sum, where if I win money, you lose that same amount of money, and vice-versa.

Imagine we have the following matrix for two players, A and B, with choices 1 and 2, from the perspective of player B.

	A1	A2
B1	10	4
B2	3	2

What is the best choice for player A and B? We analyze it on a player-by-player basis.

First suppose we are player A. If B chooses 1, A should choose 2, since they'd rather lose 4 dollars instead of 10. If B chooses 2, A should choose 2, since they'd rather lose 2 dollars instead of 3.

Now suppose we are player B. If A chooses 1, B should choose 1, since they'd rather win 10 dollars instead of 3. If A chooses 2, B should choose 1, since they'd rather win 4 dollars instead of 2.

In this case, we have an overlap, where A chooses 2, and B chooses 1. In this case, both players will be as happy as they can be. This is called a **saddle point**.

3 Mixed Strategies

But there are some situations where A and B cannot both be happy with their choices at the same time. In these cases, we need to find a mixed-strategy, where the players choose each choice with a certain probability, p for choice 1, and $(1 - p)$ for choice 2.

Image we have the following matrix for two players, A and B, with choices 1 and 2, from the perspective of player B.

	A1	A2
B1	5	2
B2	-3	4

There is no overlap in this game where both players will be happy. So we must find a mixed strategy. We will find one for player A.

If B always chooses 1, then A can either lose 5 or lose 2. If A chooses 1 with probability p, the expected value will be

$$-5p + -2(1-p)$$

If B always chooses 2, then A can either win 3 or lose 4. If A chooses 1 with probability p, the expected value will be

$$3p + -4(1-p)$$

By setting these two equations equal to each other, we can solve for p which will maximize the expected value.

$$\begin{aligned} 3p + -4(1-p) &= -5p + -2(1-p) \\ 3p + -4 + 4p &= -5p + -2 + 2p \\ 7p + -4 &= -3p + -2 \\ 10p &= 2 \\ p &= \frac{1}{5} \end{aligned}$$

So A should choose option 1 with a probability of $\frac{1}{5}$.
The expected value of this for A will be

$$-3(\frac{1}{5}) + -2$$
$$\frac{-3}{5} + \frac{-10}{5}$$
$$\frac{-13}{5}$$

So this game is a bad deal for player A.

4 Study Problems

4.1

Prove $A \Rightarrow B$ is true given the following premises:

$$\neg C \vee \neg K$$
$$Q \Rightarrow B$$
$$(\neg B \wedge A) \Rightarrow \neg B$$
$$\neg Y \vee Q$$
$$Y$$

4.2

Imagine we have the following matrix for two players, A and B, with choices 1 and 2, from the perspective of player B. Is there a saddle point? If not,

	A1	A2
B1	10	-2
B2	-5	1

what is probability that A should choose option 1?