

Math 105: Finite Mathematics

Study Problems

February 24, 2008

1 Logical Formulas

Prove whether the following statements are Tautologies, Contradictions or Neither.

$$(P \Rightarrow Q) \vee (P \wedge \neg Q)$$

$$(R \wedge S) \wedge (S \vee \neg R)$$

$$(A \wedge B) \wedge \neg A$$

2 Set Theory

Given the following sets:

$$S = \{yellow, green, blue\}.$$

$$T = \{red, green, blue\}.$$

$$X = \{purple, orange, yellow\}.$$

$$Y = \{black, brown, white, blue\}.$$

$$U = \{yellow, green, red, purple, orange, black, brown, white, blue\}.$$

where U is the set of all elements. What are the elements in the sets below?

$$S \cup T.$$

$$S \cap T.$$

$$X' \cap T'.$$

$$T \cup Y.$$

3 Counting Sets

Your aquarium has 15 fish total. 9 fish are orange, and 7 are small, while 3 are neither small nor orange. How many are both small and orange? Draw a Venn diagram and label each subset with its size.

The pet store has 4 varieties of fish tanks, 5 types of gravel, and 3 kinds of habitats (sea diver, treasure chest, and pirate ship). How many unique ways are there to make a fish tank?

4 Basic Probability

What is the probability that the sum of rolling two six-sided die will be a prime number? (prime numbers are divisible only by 1 and themselves).

The probability of a freezer containing ice cream is 90%. The probability of getting the flu is 10%. If we assume these probabilities are independent, what is the probability of getting the flu while there is ice cream in your freezer?

The probability of smiling when watching a movie is 40%. The probability of frowning is 15%. If these events are mutually exclusive, what is the probability of either smiling or frowning?

The probability of having your credit information stolen during each eBay order is 0.003. If you order 245 items through eBay, what is the probability of having your credit information at least once?

5 Combinatorics

20 people work for the local firing squad. How many ways are there to select a team of 9 people and put them into a unique ordering?

You are studying birds on a remote island, and know there are 300 birds on this island. You have managed over the past few months of research to band the legs of 135. If you randomly choose 14 birds, what is the probability that exactly 5 of them are banded?

What is the probability of seeing exactly 10 red cars out of 45, if the probability of seeing a red car by chance is 20%?

6 Calibration

You find a 7-sided coin, but one side is chipped, so instead of each side being equally likely, the number 3 comes up 10% more than the other sides. What

is the probability for each side? You decide to roll the die 5 times, and get the results of {6, 2, 3, 4, 4}. What value did you actually roll? What is the probability for each side after rolling the die 5 times?

7 Conditional Probability and Bayes' Rule

A movie has a 5% chance of winning Best Picture, and a 10% chance of winning Best Director. If a movie wins Best Director, the probability of winning Best Picture is 45%. If you only know a movie has just won Best Picture, what is the chance it also won Best Director?