Rate of Change

Chapter 1

Section 2

Sale of DVDs and VCRs

Sales of DVD players have been increasing since they were introduced in 1998, while the sales of VCRs have been decreasing. See the data in the table below. Sales are listed in millions of $.

Year	1998	1999	2000	2001	2002	2003
VCR sales	2409	2333	1869	1058	826	407
DVD player sales	421	1099	1717	2097	2427	3050

Calculating Rate of Change

Let us calculate the rate of change of DVD player and VCR sales between 1998 and 2003.

$$\text{rate of change} = \frac{\text{change in sales}}{\text{change in time}}$$

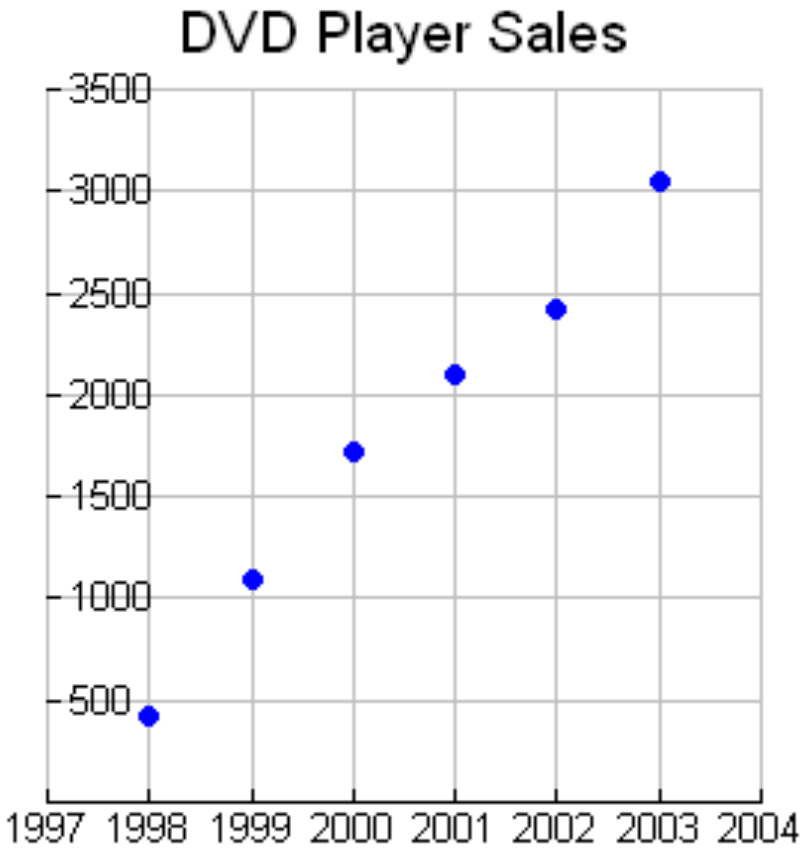
Year	1998	1999	2000	2001	2002	2003
VCR sales	2409	2333	1869	1058	826	407
DVD player sales	421	1099	1717	2097	2427	3050

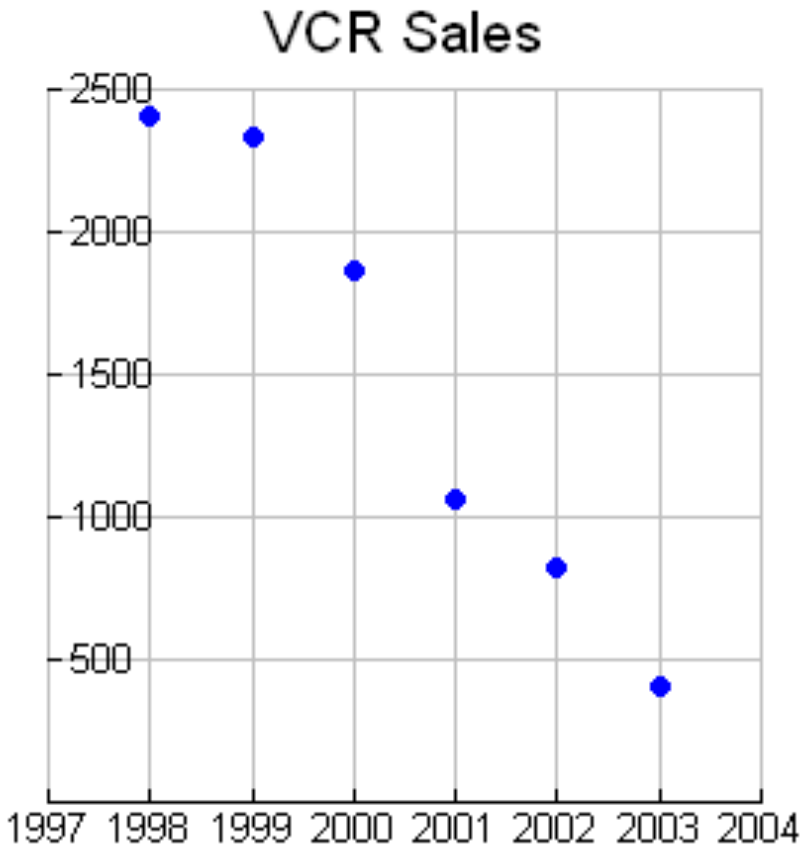
Graphing the Sales Data

421	1099	1717	2097	2427	3050

2409	2333	1869	1058	826	407

Graphing the Sales Data





Rate of Change of a Function

- Suppose that Q is a function of t, i.e. $Q = f(t)$.
- The ***average rate of change***, or ***rate of change***, of Q with respect to t over an interval is given by the formula below.

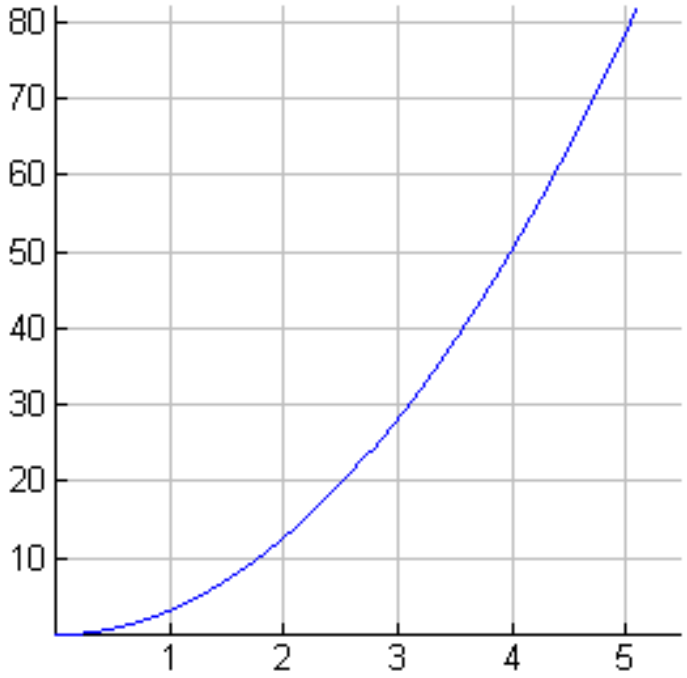
$$\text{Average rate of change over an interval} = \frac{\text{Change in } Q}{\text{Change in } t} = \frac{\Delta Q}{\Delta t}$$

Increasing and Decreasing Functions

- If $Q = f(t)$ for t in the interval $a \leq t \leq b$.
  - f is an ***increasing function*** if the values of f increase as t increases in this interval.
  - f is a ***decreasing function*** if the values of f decrease as t increases in this interval.
- If $Q = f(t)$
  - If f is an ***increasing function***, then the average rate of change of Q with respect to t is positive.
  - If f is a ***decreasing function***, then the average rate of change of Q with respect to t is negative.

Example – Area of a Circle

The function $A = q(r) = \pi r^2$, gives the area, A, of a circle as a function of its radius, r. Graph q. Is the function q increasing or decreasing?

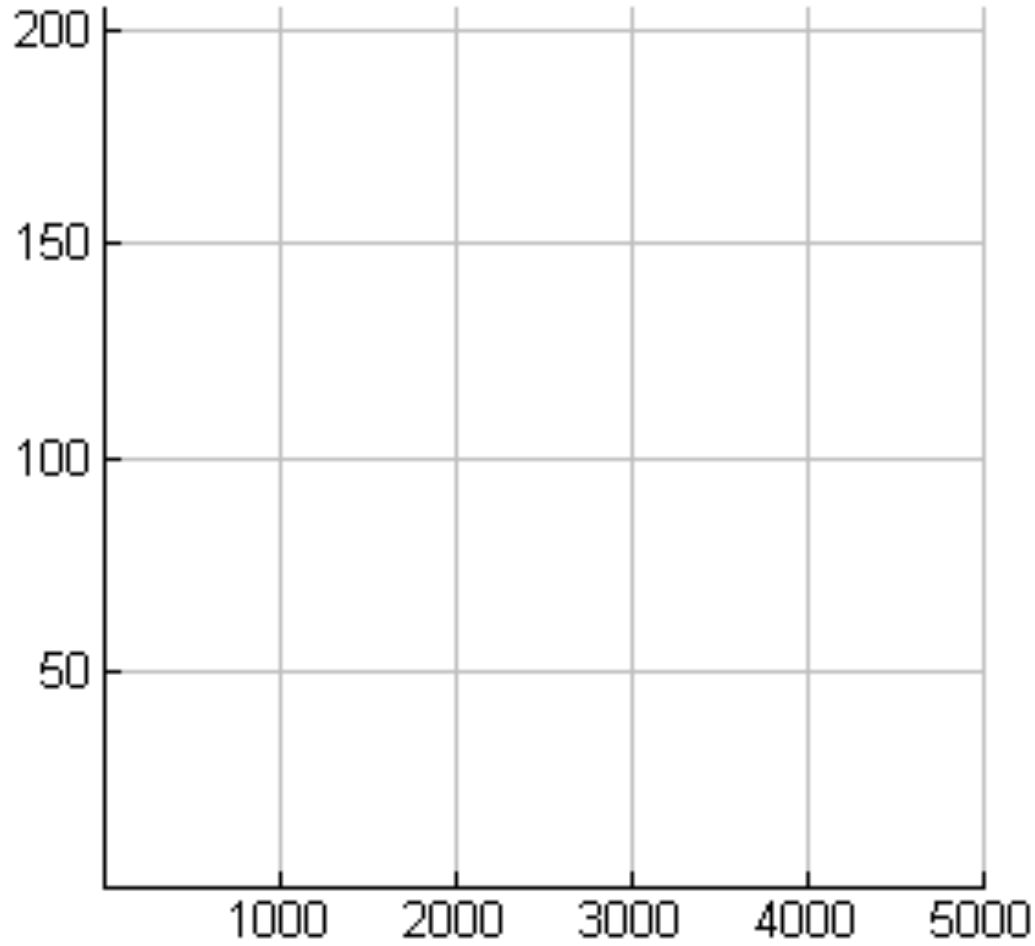


Example – Carbon-14

Carbon-14 is a radioactive element that exists naturally in the atmosphere and is absorbed by living organisms. When an organism dies, the carbon-14 present at death begins to decay. Let $L = g(t)$ represent the quantity of carbon-14 in a tree t years after its death. See the data listed below. Why is g decreasing? Graph g.

t, time (years)	0	1000	2000	3000	4000	5000
L, carbon -14	200	177	157	139	123	109

Carbon-14 Graph



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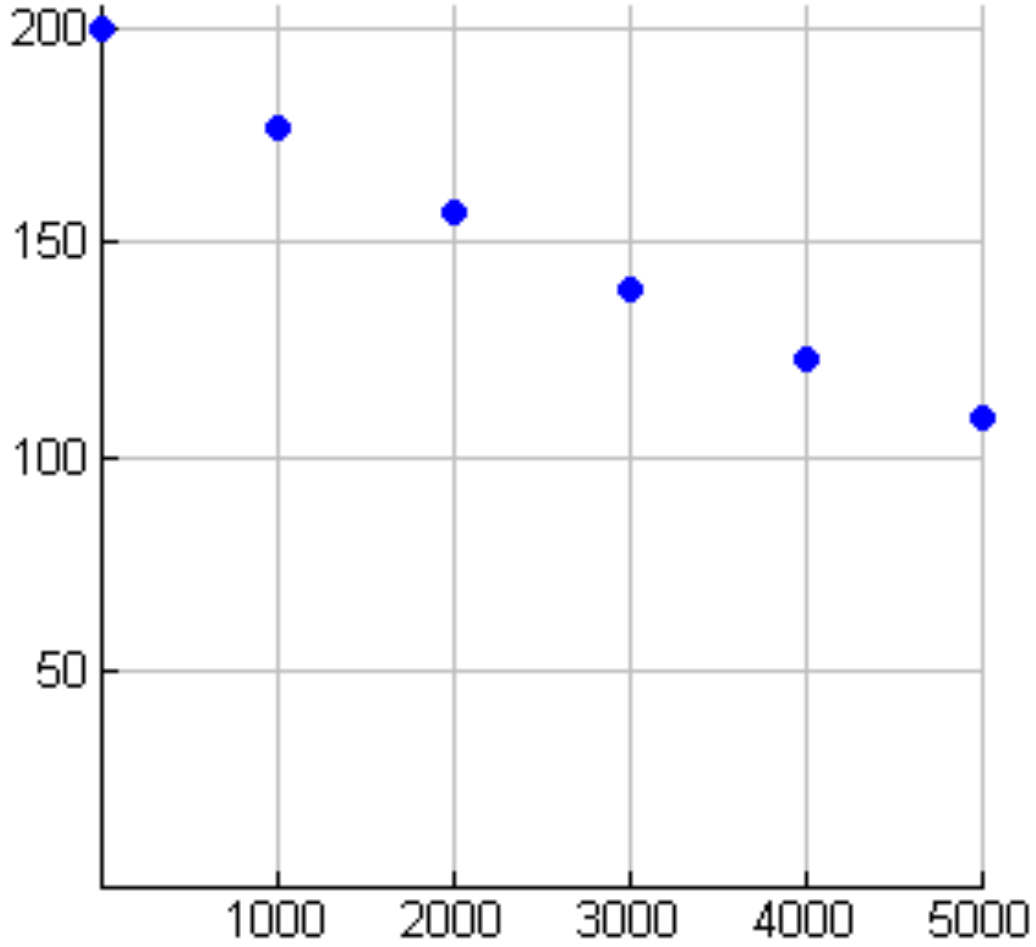
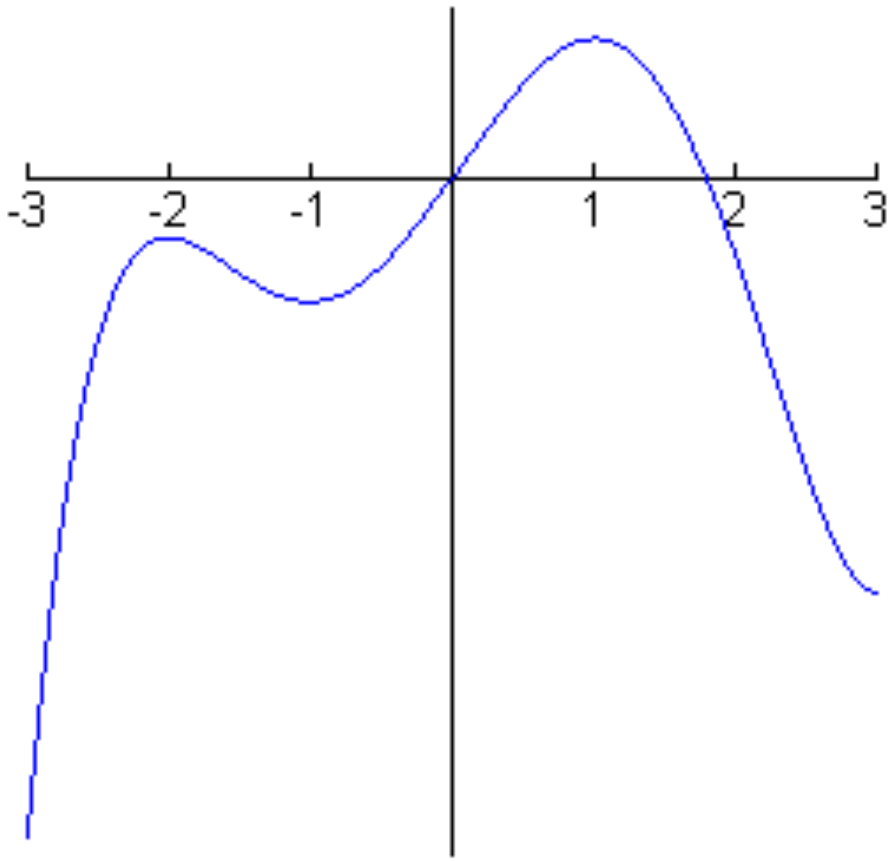
Carbon-14 Graph



t, time (years)	0	1000	2000	3000	4000	5000
L, carbon -14	200	177	157	139	123	109

Graphing Example

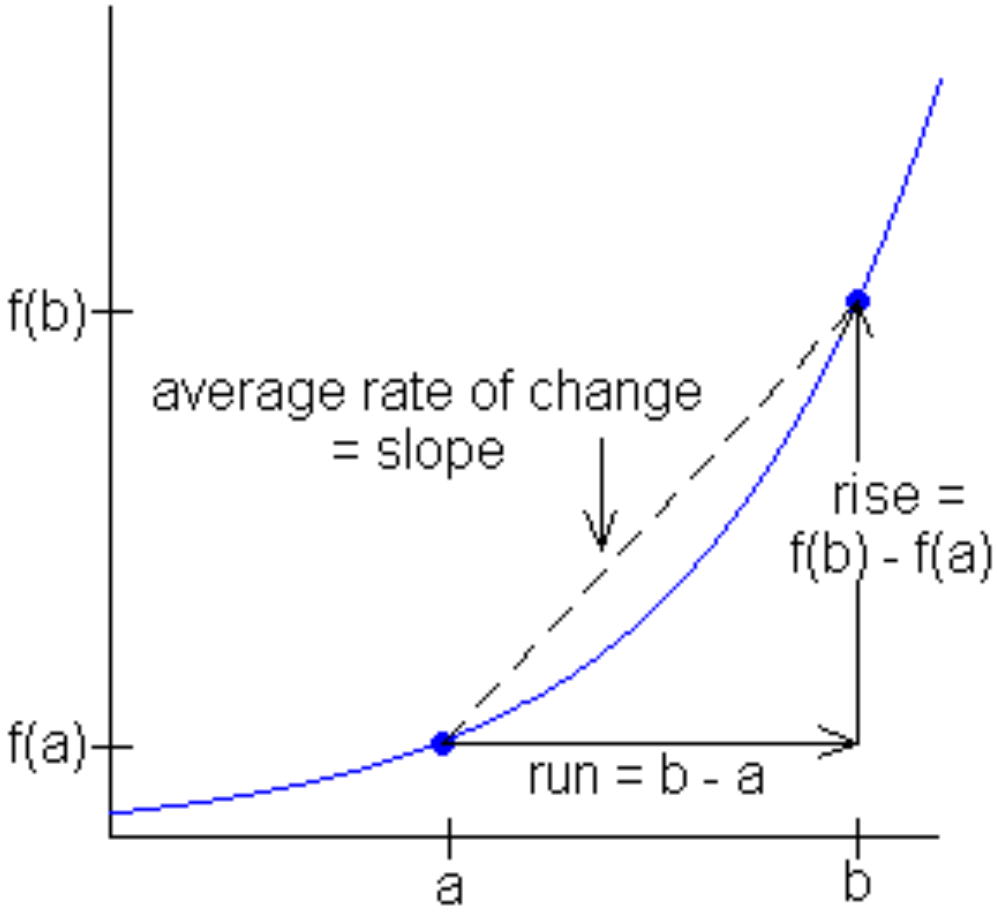
On what intervals is the function graphed to the right increasing? Decreasing?

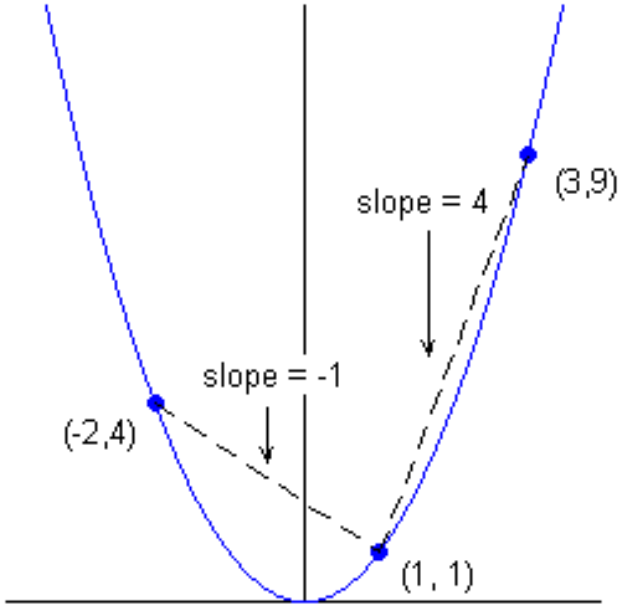


Function Notation for the Average Rate of Change

Let $Q = f(t)$ on the interval $a \le t \le b$

$$\text{Average rate of change} = \frac{\text{Change in } Q}{\text{Change in } t} = \frac{\Delta Q}{\Delta t} = \frac{f(b) - f(a)}{b - a}$$



Example – Rates of Change from a Formula

Calculate the average rates of change of the function $f(x) = x^2$ between $x = 1$ and $x = 3$ and between $x = -2$ and $x = 1$. Show your results on a graph.