Linear Functions

Chapter 1

Section 3

Constant Rate of Change

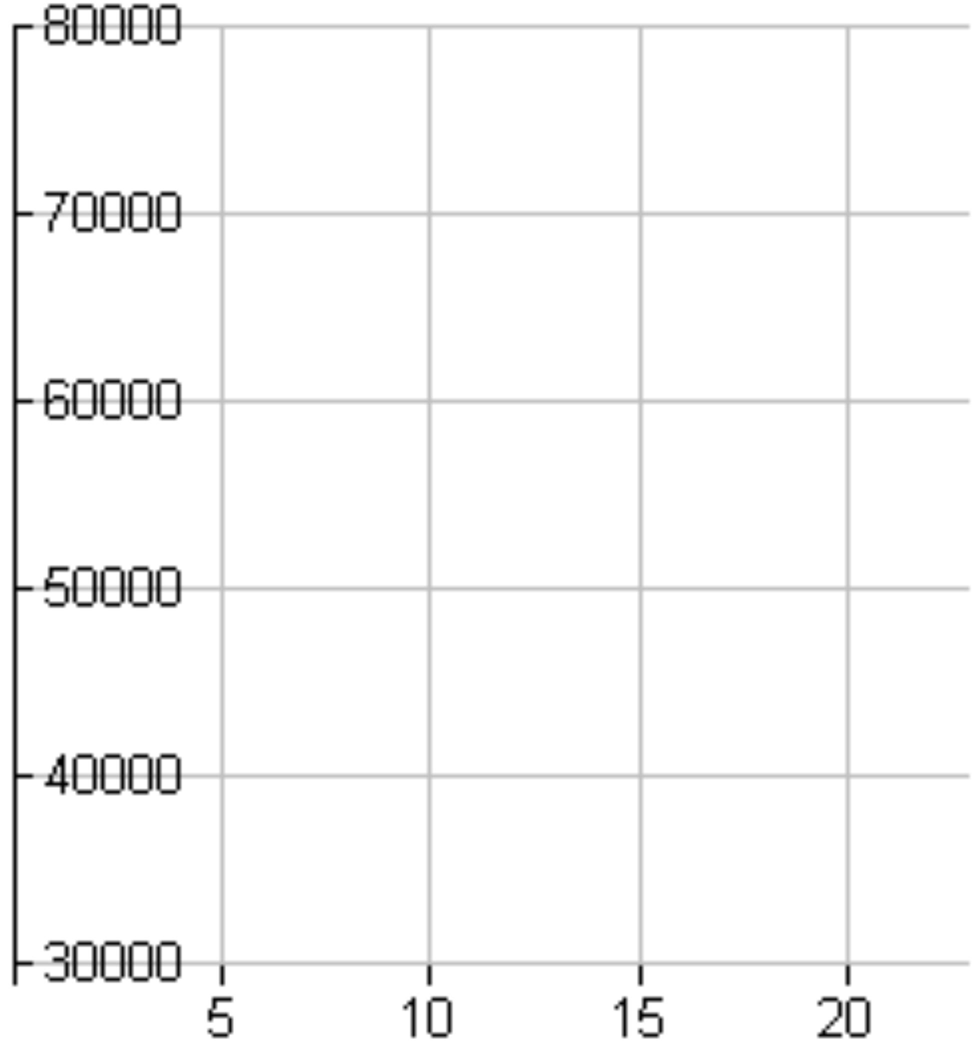
In the remaining sections, 3, 4, 5 and 6, of this chapter we will consider functions which have the same average rate of change on every interval. Functions which have the same or a constant rate of change have a graph that is a line and hence are called ***linear***.

Example – Population Growth

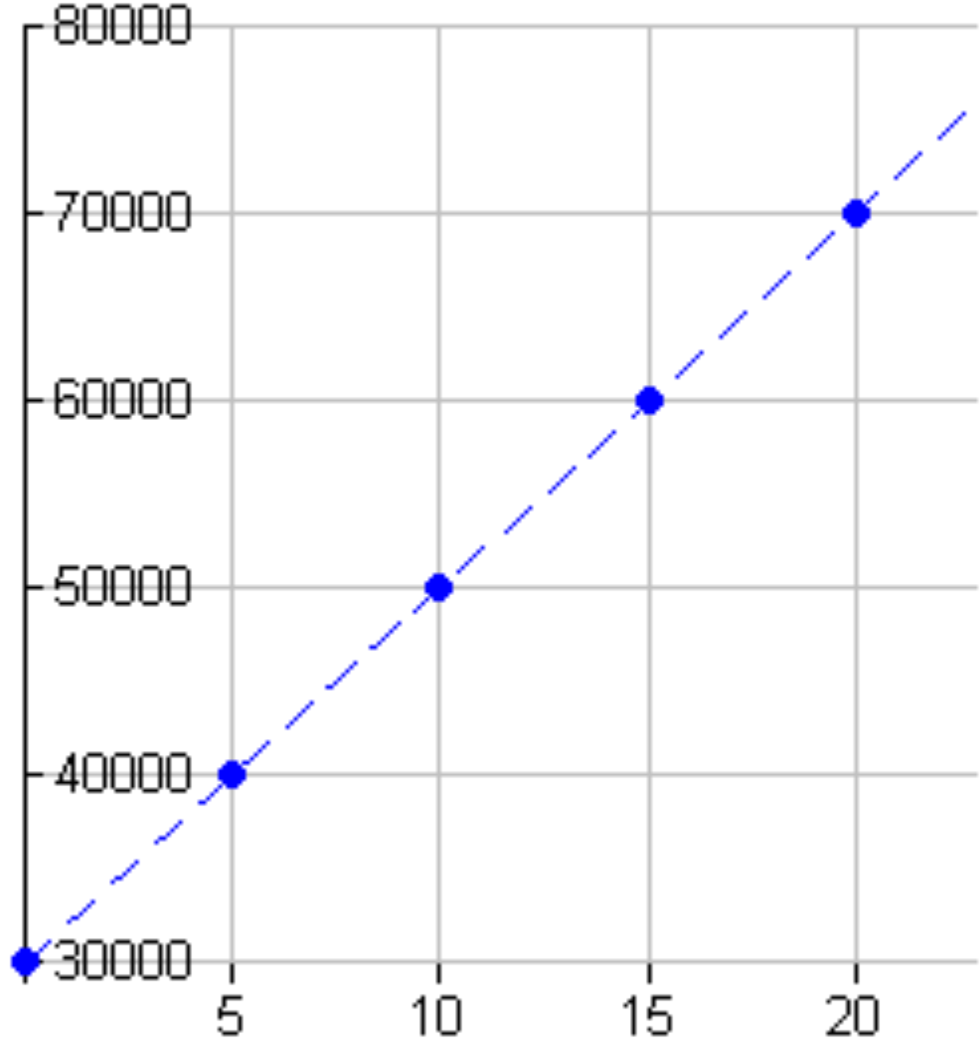
A town of 30,000 people grows by 2,000 people every year. Since the population, P, is growing at the constant rate of 2,000 people per year, *P*, is a linear function of time, *t*, in years.

a) What is the average rate of change of *P* over every time interval?
b) Make a table that gives the town's population every five years over a 20-year period. Graph the population.
c) Find a formula for *P* as a function of *t.*

t	*P*
0	30,000
5	
10	
15	
20	



t	*P*
0	30,000
5	40,000
10	50,000
15	60,000
20	70,000



Characteristics of a Linear Function

- A **linear function** has a constant rate of change.
- The graph of any linear function is a straight line.

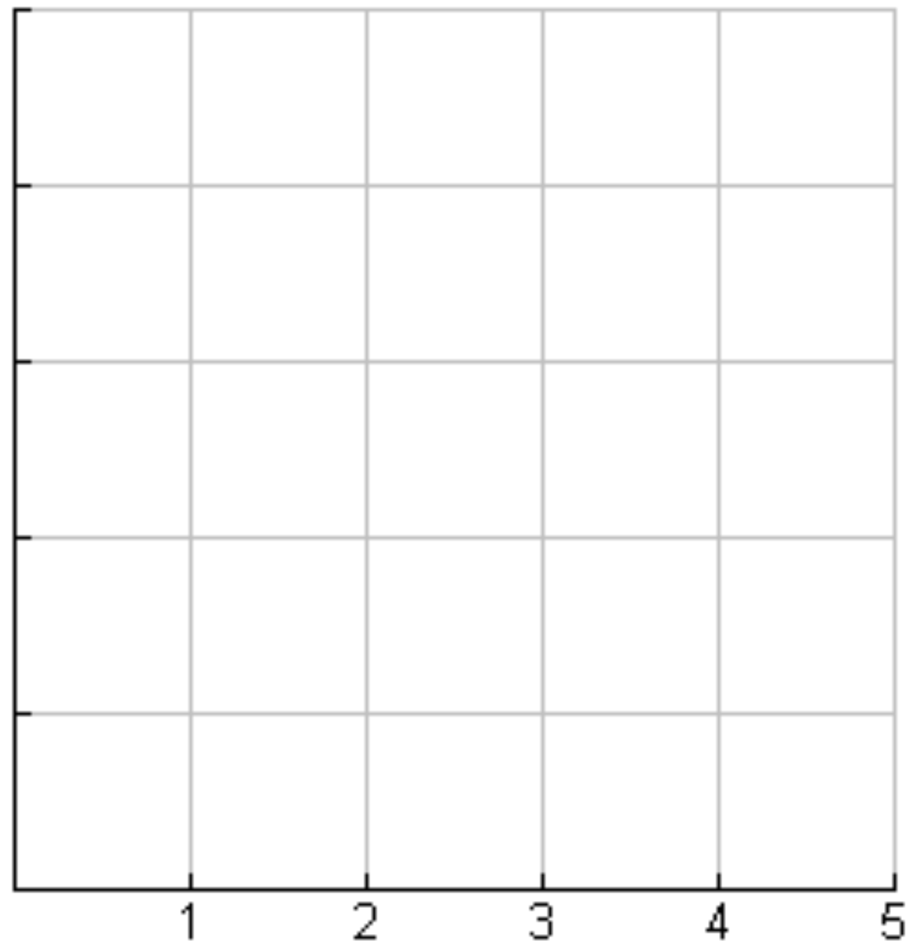
Example – Financial Models

A small business spends \$20,000 on new computer equipment and, for tax purposes, chooses to depreciate it to \$0 at a constant rate over a five-year period.

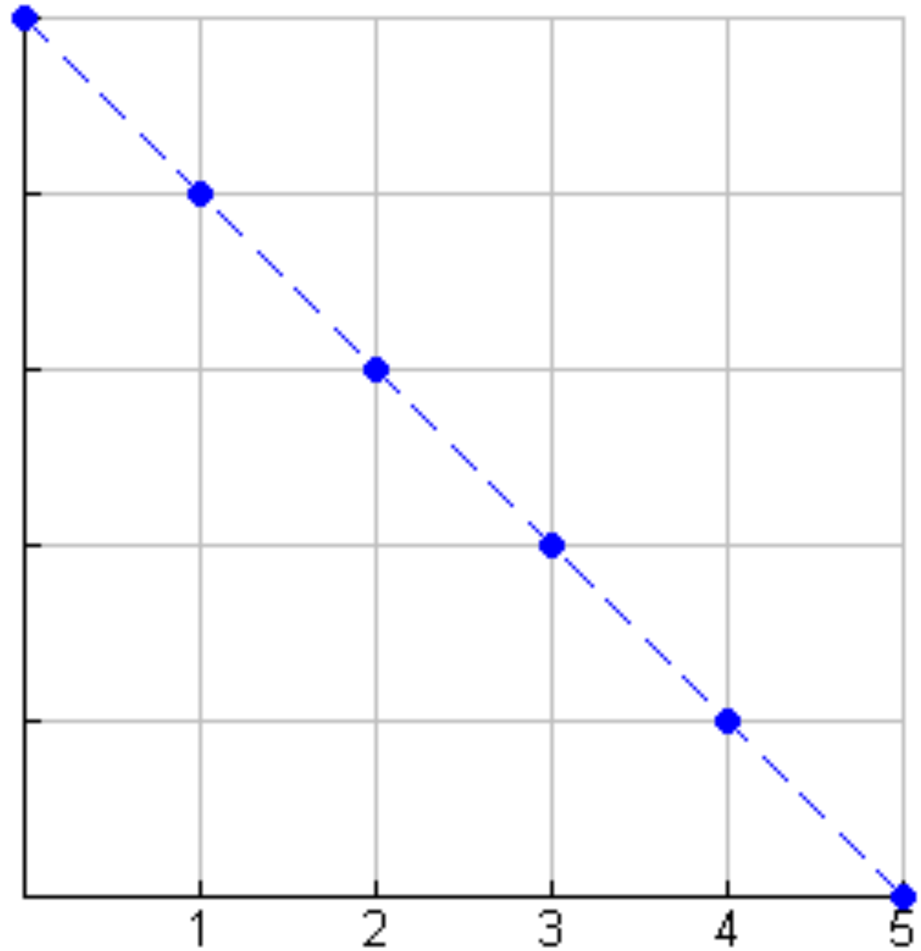
a) Make a table and a graph showing the value of the equipment over the five-year period.

b) Give a formula for value as a function of time.

t	V
0	20,000
1	
2	
3	
4	
5	0



t	V
0	20,000
1	16,000
2	12,000
3	8,000
4	4,000
5	0



Equation for Population Growth

In the population growth example, the equation for the function was Current population = Initial population + Growth rate per year × Number of years or $P = 30{,}000 + 2{,}000t$.

$$\text{Current population} = \underbrace{\text{Initial population}}_{30{,}000} + \underbrace{\text{Growth rate}}_{2{,}000} \times \underbrace{\text{Number of years}}_{t}$$

Equation for Business Equipment

In the business equipment example, the equation for the function was Current value = Initial value – Rate of change per year × Number of years or $V = 20{,}000 - 4{,}000t$.

$$\text{Total cost} = \underbrace{\text{Initial value}}_{20{,}000} + \underbrace{\text{Change per year}}_{-4{,}000} \times \underbrace{\text{Number of years}}_{t}$$

General Formula for a Linear Function

Using the symbols *x, y, b, m,* we see formula for both of the previous examples follows the same pattern:

$$\underbrace{\text{Output}}_{y} = \underbrace{\text{Initial value}}_{b} + \underbrace{\text{Rate of change}}_{m} \times \underbrace{\text{Input}}_{x}$$

$$y = b + mx$$

Slope-Intercept Equation for a Linear Function

If $y = f(x)$ is a linear function, then for some constants b and m: $y = b + mx$.

- m is the ***slope*** and gives the rate of change of y with respect to x. The slope tells you how much y changes for a one-unit change in x.
- b is the ***vertical intercept***, or ***y-intercept***, and gives the value of y when $x = 0$. In mathematical models, b typically represents an initial, or starting value of the output.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Summary of Linear Notation

- In the population growth example the population function, $P = 30{,}000 + 2{,}000t$, has slope $m = 2{,}000$ and vertical intercept $b = 30{,}000$.
- In the business equipment example the depreciation function, $V = 20{,}000 – 4{,}000t$, has slope $m = -4{,}000$ and vertical intercept $b = 20{,}000$.

Tables for Linear Functions

The table shown below gives values of two function, p and q. Could either of these function be linear?

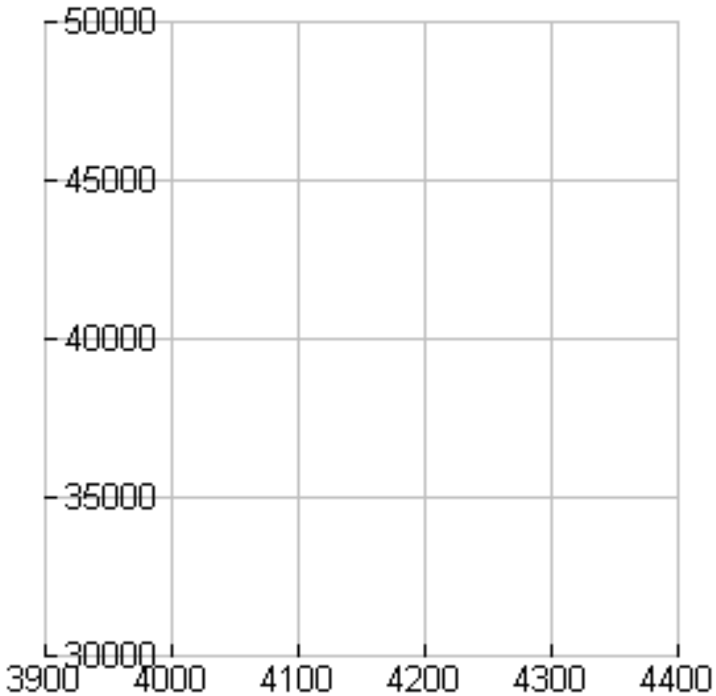
x	50	55	60	65	70
$p(x)$	0.10	0.11	0.12	0.13	0.14
$q(x)$	0.01	0.03	0.06	0.14	0.15

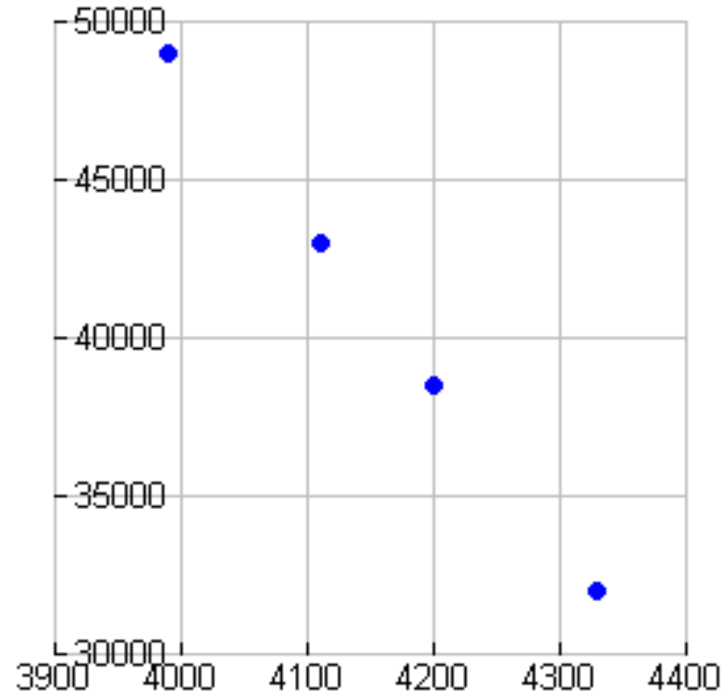
Yugo Example

Year	1985	1986	1987	1988
Price, *p*	3990	4110	4200	4330
# sold, *Q*	49000	43000	38500	32000

1) Using the table above explain why *Q could be a linear function of* p.
2) What does the rate of change of this function tell you about Yugos?

Year	1985	1986	1987	1988
Price, *p*	3990	4110	4200	4330
# sold, *Q*	49000	43000	38500	32000

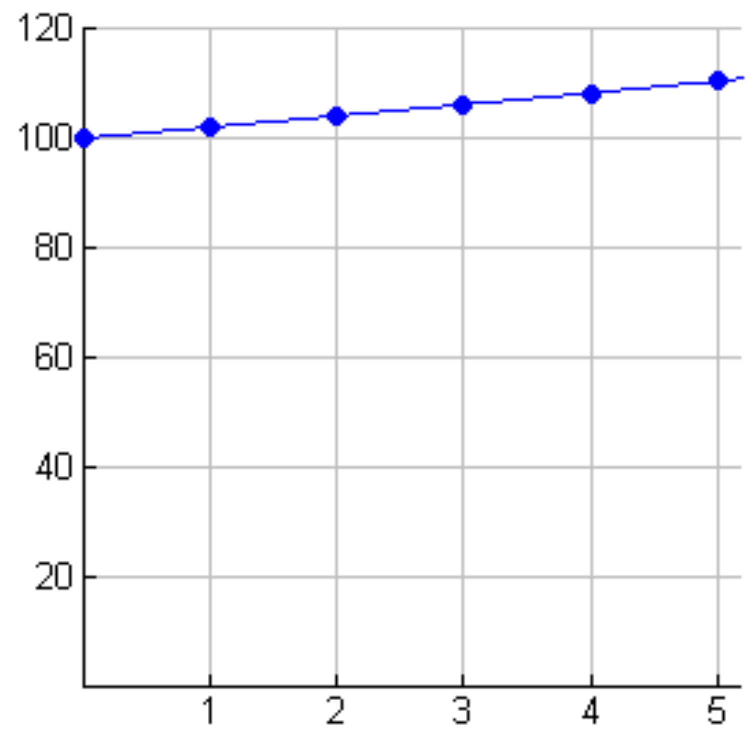




Looks Can Be Deceiving

Population of Mexico is given in the table and graph given below. The data appears to be linear.

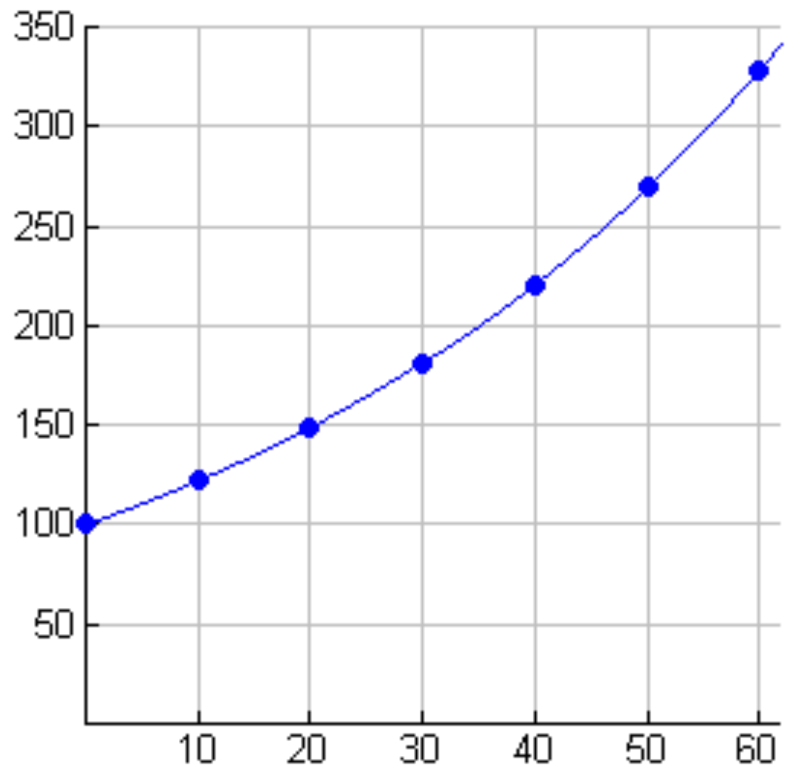
t (years)	*P* (millions)
0	100
1	102
2	104.04
3	106.12
4	108.24
5	110.41



Population of Mexico over 60 Years

The formula used to calculate the population of Mexico is $P = 100\ (1.02)^x$.

t (years)	*P* (millions)
0	100
10	121.90
20	148.59
30	181.14
40	220.80
50	269.16
60	328.10



Problem #14

- In 2006, the population of a town was 18,310 and growing by 58 people per year. Find a formula for P, the town's population, in terms of *t*, the number of years since 2006.

Problem #17

The table below shows the cost *C*, in dollars, of selling *x* cups of coffee per day from a cart.

a) Show this is a linear relationship.

b) Plot the data in the table.

c) Find the slope of the line. Explain what this means in the context of the given situation.

d) Why might it cost $50 to serve zero cups of coffee?

x	0	5	10	50	100	200
C	50	51.25	52.50	62.50	75.00	100.00