Comparing Exponential and Linear Functions

Chapter 4

Section 2

Identifying Linear and Exponential Functions From a Table

The table below gives values of a linear and an exponential function. Decide which is which.

x	20	25	30	35	40	45
$f(x)$	30	45	60	75	90	105
$g(X)$	1000	1200	1440	1728	2073.6	2488.32

Identification Summary

For a table of data that gives *y* as a function of *x* and in which Δx is constant:

1) If the ***difference*** of consecutive *y*-values is constant, the table could represent a linear function.

2) If the ***ratio*** of consecutive *y*-values is constant, the table could represent an exponential function.

Finding a Formula for an Exponential Function

x	20	25	30	35	40	45
$g(X)$	1000	1200	1440	1728	2073.6	2488.32

The general form for an exponential equation is $g(x) = ab^x$. The table tells us that $ab^{20} = 1000$ and $ab^{25} = 1200$.

Modeling Linear and Exponential Growth Using Two Data Points

At time t = 0 years, a species of turtle is released into a wetland. When t = 4 years, a biologist estimates there are 300 turtles in the wetland. Three years later, the biologist estimates there are 450 turtles. Let P represent the size of the turtle population in year t.

Modeling Linear and Exponential Growth Using Two Data Points

Using $P(4) = 300$ and $P(7) = 450$:

1) Find a formula for $P = f(t)$ assuming linear growth. Interpret the slope and P-intercept of your formula in terms of turtle population.
2) Find a formula for $P = g(t)$ assuming exponential growth. Interpret the parameters of your formula in terms of turtle population.
3) In year $t = 12$, the biologist estimates that there are 900 turtles in the wetland. What does this indicate about the two population models

Similarities and Differences between Linear and Exponential Functions

$$y = b + mx$$

$$y = b + \underbrace{m + m + m + \cdots + m}_{x \text{ times}}$$

$$y = ab^x$$

$$y = a \cdot \underbrace{b \cdot b \cdot b \cdot \ldots \cdot b}_{x \text{ times}}$$

Linear vs. Exponential

For each of the following tables, decide if the function is linear or exponential and find a possible formula for the function.

x	$f(x)$
0	65
1	75
2	85
3	95
4	105

x	$g(x)$
0	400
1	600
2	900
3	1350
4	2025

Exponential vs. Linear Growth

It can be shown that an exponentially increasing quantity will, in the long run, always outpace a linearly increasing quantity. This fact led the 19th-century clergyman and economist, Thomas Malthus, to make some rather gloomy predictions, which are illustrated in the next slide.

Population vs. Food Supply Example

The population of a country is initially 2 million and is increasing at 4% per year. The food supply is initially adequate for 4 million and is increasing at a constant rate of 0.5 million.

1) Based on these assumptions, when will the population exceed food supply?
2) If the country doubled its initial food supply, would shortages still occur? If so, when?
3) What happens if both the rate and the initial food supply double?

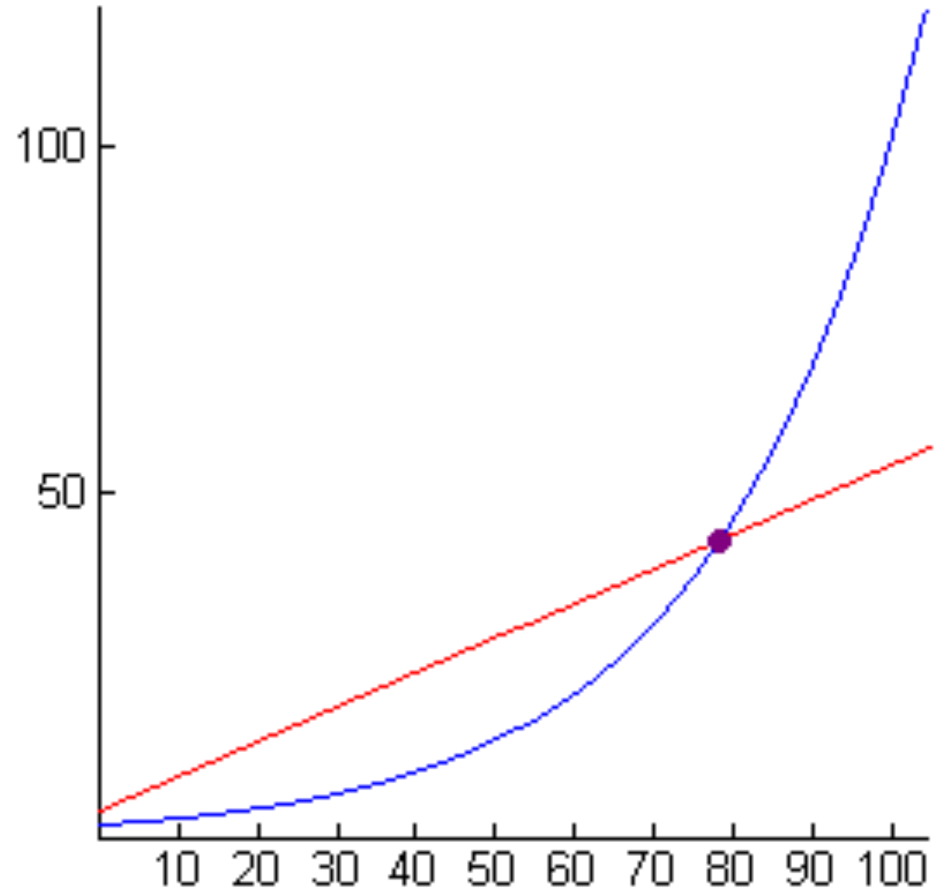
Answer to Question 1

Population equation

$$P = 2 \cdot 1.04^t$$

Food supply equation

$$F = 4 + 0.5t$$



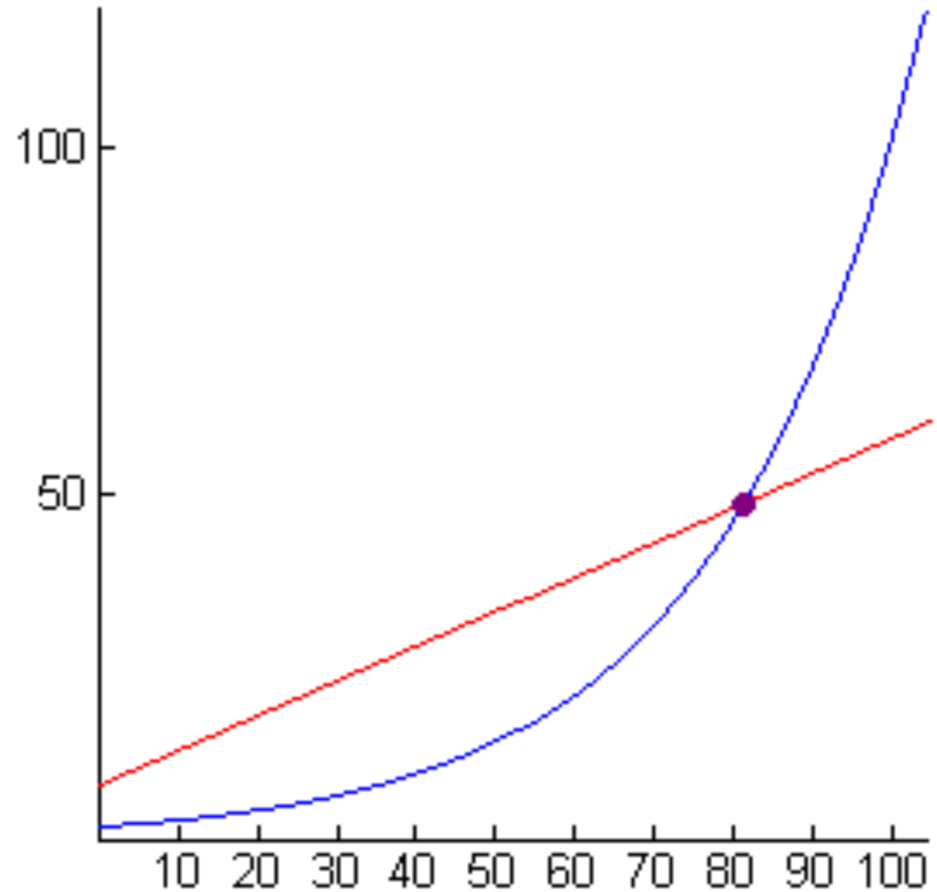
Answer to Question 2

Population equation

$$P = 2 \cdot 1.04^t$$

Food supply equation

$$F = 8 + 0.5t$$



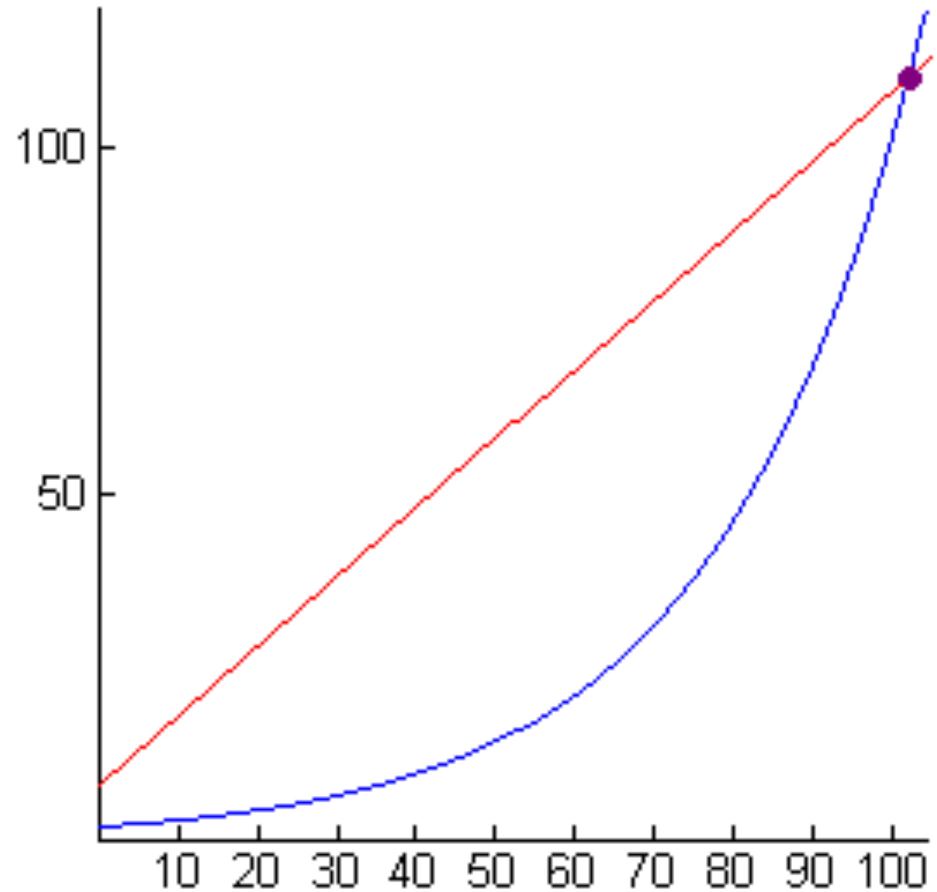
Answer to Question 3

Population equation

$$P = 2 \cdot 1.04^t$$

Food supply equation

$$F = 8 + t$$



Exercise #24

Consider the data in the table to the right.

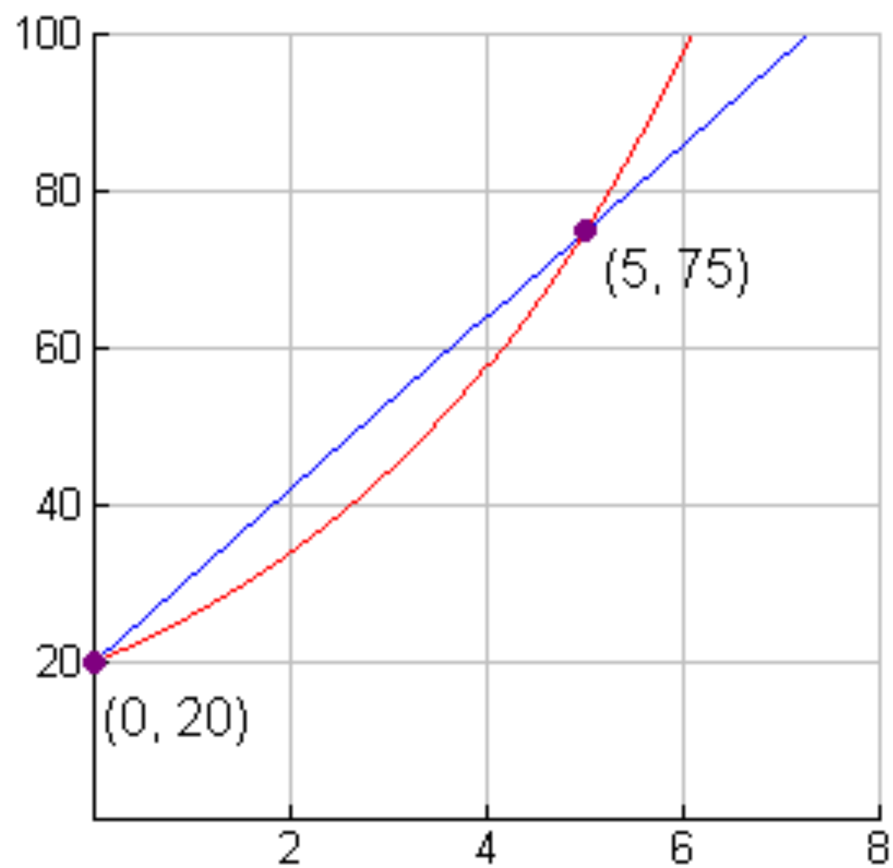
a) Decide is the function is linear or exponential.

b) Find a possible formula for the function.

x	$h(x)$
0	14
1	12.6
2	11.34
3	10.206
4	9.185

Problem #26

Graphs of a linear and an exponential function are shown in the figure to the right. Find formulas for each of the functions.



Problem #37

Short track 500m speed skating became a Winter Olympic event in 1994, and Chae Ji-Hon of Korea won the event that year with a time of 43.45 seconds. In 2006, Apolo Ohno of the US won the event with a time of 41.94 seconds. Find a formula for the predicted winning time in the 500m speed skating event as a function of the number of years since 1994, and predict the winning time in 2018, if we assume the decrease in time is a linear function or is an exponential function.

Problem #7

Suppose that $f(x)$ is exponential and that $f(-3) = 54$ and $f(2) = 2/9$. Find a formula for $f(x)$.

Problem #15

Find a formula for the exponential function pictured in the graph to the right.