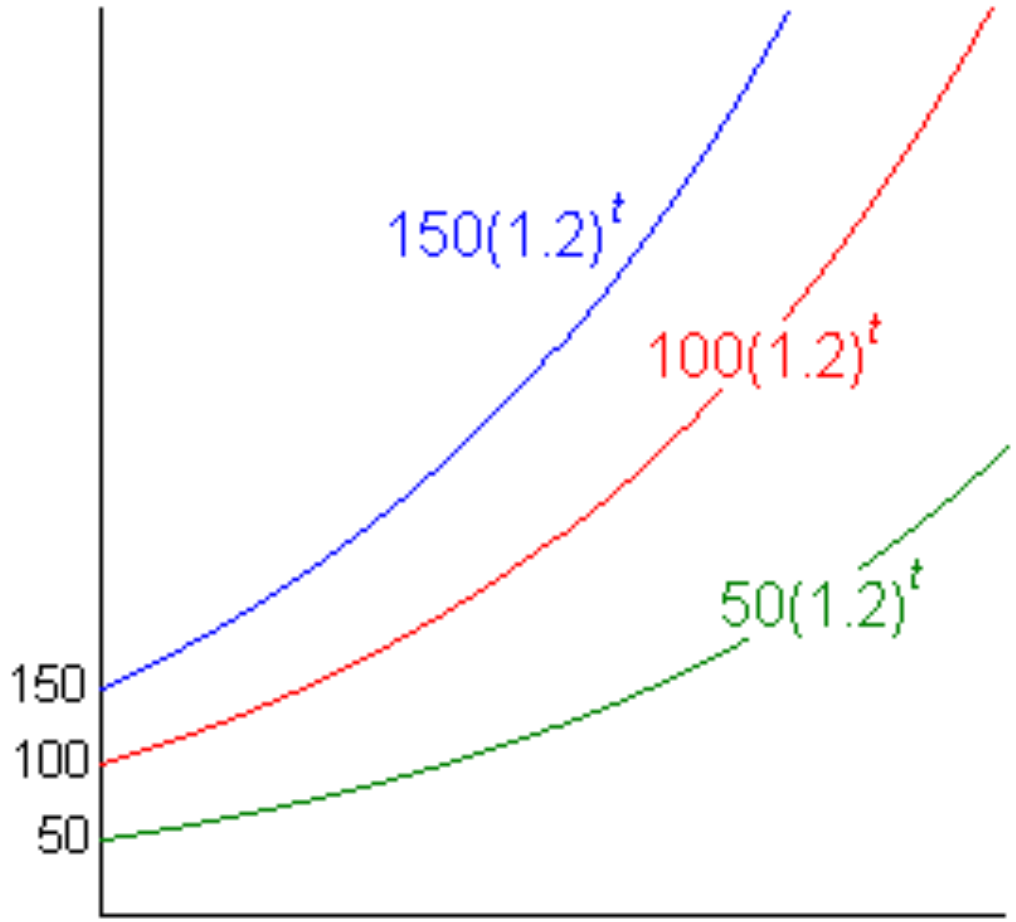
Graphs of Exponential Functions

Chapter 4

Section 3

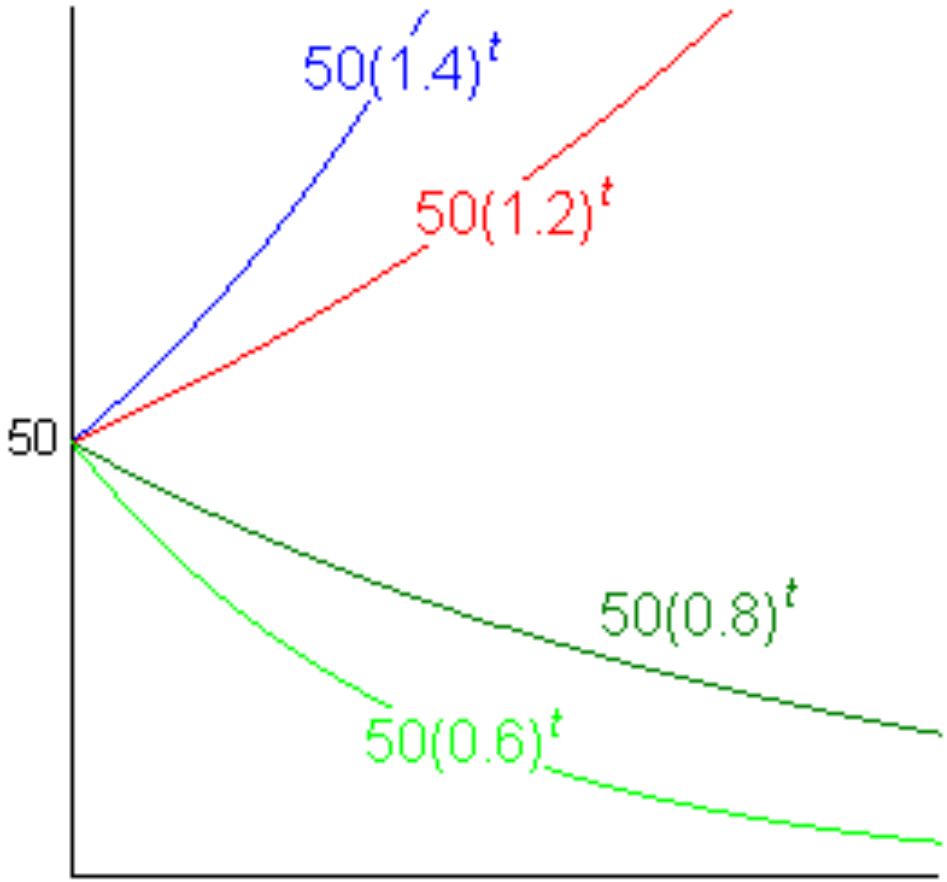
The Effect of the Parameter *a*

In the formula $Q = ab^t$, the value of a tells us where the graph crosses the Q-axis. Since a is the value of Q when $t = 0$.



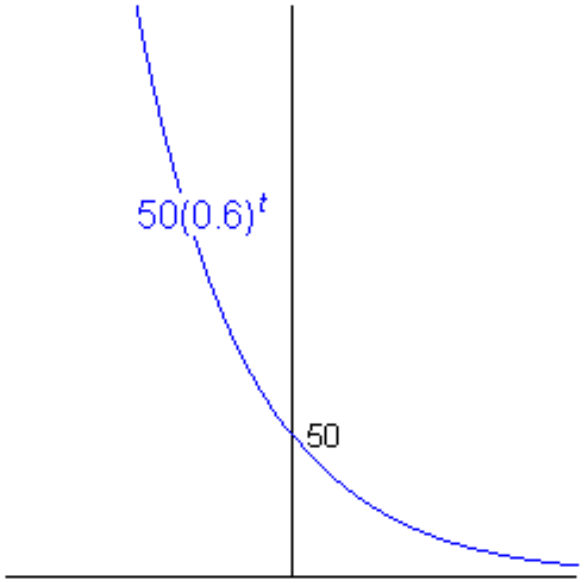
The Effect of the Parameter *b*

In the formula $Q = ab^t$, the value of b is the growth factor or base and tells us how steep the graph is.



Horizontal Asymptotes

The *t*-axis is a ***horizontal asymptote*** for the graph $Q = ab^t$, because Q approaches 0 as t gets large, either positively or negatively.



$$Q \to 0 \text{ as } t \to \infty$$

$$\lim_{t \to \infty} f(t) = 0$$

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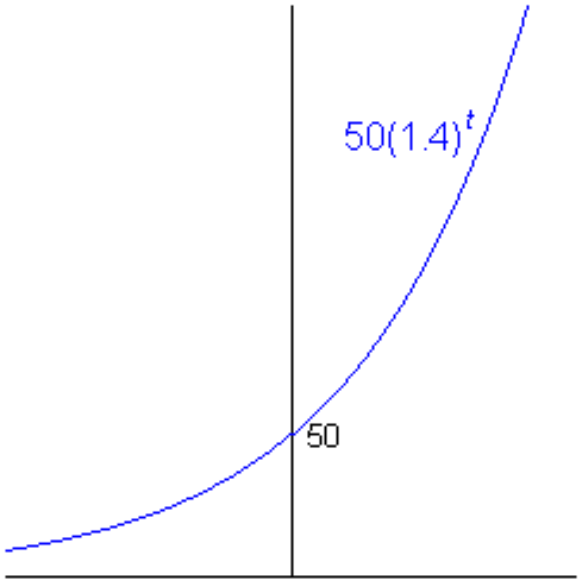


$$Q \to 0 \text{ as } t \to -\infty$$

$$\lim_{t \to -\infty} f(t) = 0$$

Summary for Horizontal Asymptote

The horizontal line $y = k$ is a ***horizontal asymptote*** of a function, f, if the function values get arbitrarily close to k as x gets large (either positively or negatively or both).

$$f(x) \to k \text{ as } x \to \infty \text{ or } f(x) \to k \text{ as } x \to -\infty$$

$$\lim_{x \to \infty} f(x) = k \text{ or } \lim_{x \to -\infty} f(x) = k$$

Capacitor Example

A capacitor is the part of an electrical circuit that stores electric charge. The quantity of charge decreases exponentially with time. Assume the charge equations is $Q = 200(0.9)^t$.

1) Describe in words how the stored charge changes over time.
2) What quantity of charge remains after 10 seconds? 20 seconds? 1 minute? 2 minutes?
3) Graph the charge over the first minute. What does the horizontal asymptote of the graph tell you about the charge?

Town of Yonkers Example

Recall the fine, P, imposed on the city of Yonkers is given by $P = 100 \cdot 2^t$. In 1988, the annual budget of the city was $337 million. When will the fine wipe out the entire city budget? Graph $P = 100 \cdot 2^t$ and $P = 337$ and see for what value of t they intersect.

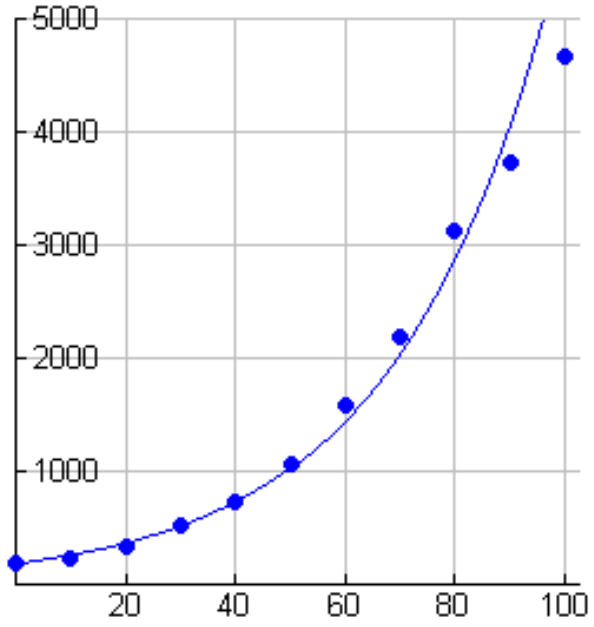
Another Carbon-14 Example

A 200 μg sample of carbon-14 decays according to the formula $Q = 200(0.886)^t$, where t is in thousands of years. Estimate when there is 25 μg of carbon-14 left by graphing.

Fitting Exponential Functions to Data

The data in the table below gives population data for the Houston Metro Area since 1900. Use a STAT PLOT to graph this data.

t	*N*	*t*	*N*
0	184	60	1583
10	236	70	2183
20	332	80	3122
30	528	90	3733
40	737	100	4672
50	1070		



Intersecting Graphs

For each of the following decide if the graphs of the functions cross in the first quadrant. Explain your reasoning without using a graphing calculator.

$$f(x) = 20 \cdot (1.05)^x \text{ and } g(x) = 50 \cdot (1.02)^x$$

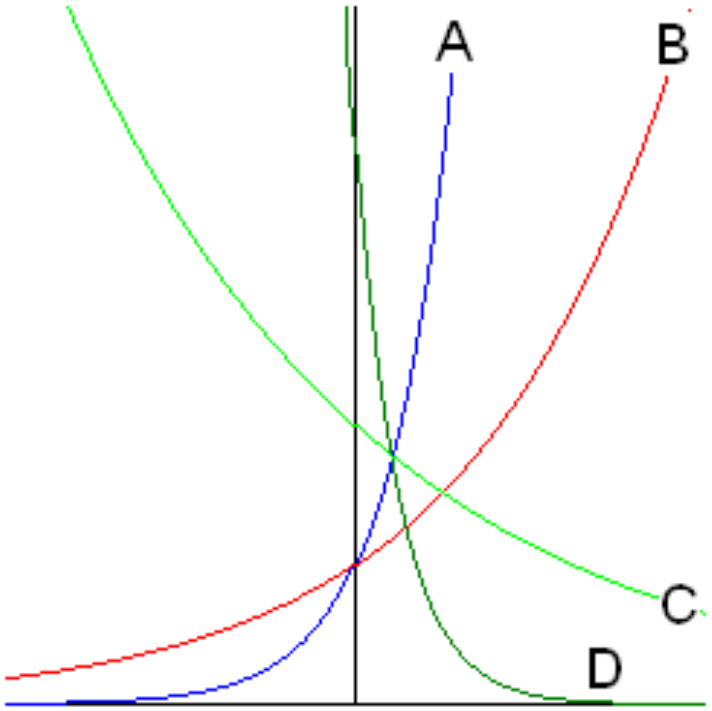
$$f(x) = 10 \cdot (1.07)^x \text{ and } g(x) = 5 \cdot (1.05)^x$$

$$f(x) = 20 \cdot (.98)^x \text{ and } g(x) = 5 \cdot (1.02)^x$$

$$f(x) = 10 \cdot (.95)^x \text{ and } g(x) = 20 \cdot (1.05)^x$$

Exercises #11-#14

Assume the equations for *A,* B, *C* and *D* can all be written in the form $y = ab^t$.
Use the graph to the right to answer the following:



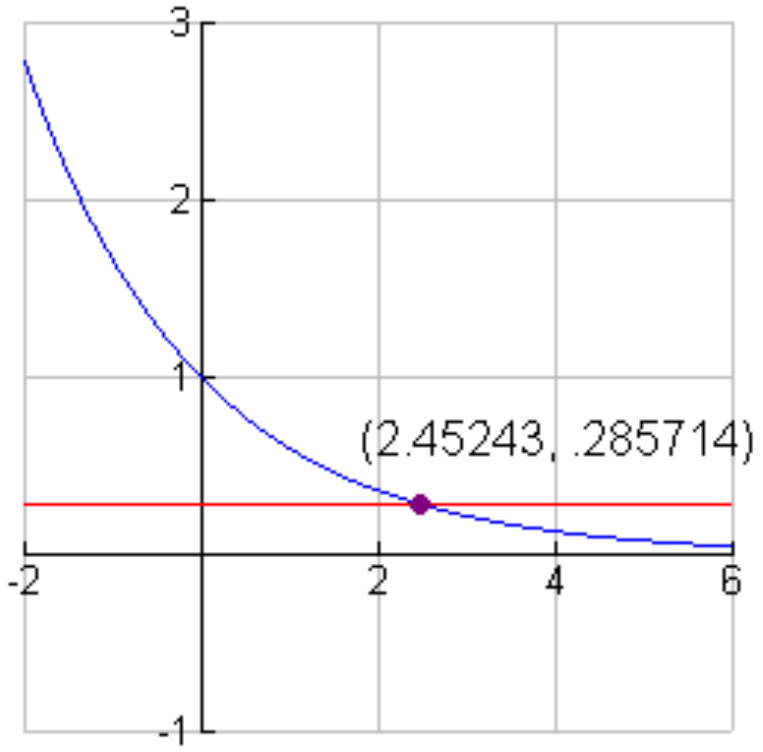
Which function has the greatest value of *b*?

Which function has the greatest value of *a*?

Exercise #19

Solve the equation listed below graphically for t if $P = 2$.

$$P/7 = (0.6)^t$$



Problems #25 and #27

Graph $f(x)$, a function defined for all real numbers and satisfying the given condition.

$$\lim_{x\to-\infty} f(x) = 2 \text{ and } \lim_{x\to\infty} f(x) = -1$$

$f(x)$ has a horizontal asymptote of $y = 5$