The Number *e*

Chapter 4

Section 5

Continuous Growth

If $1.00 is invested in a bank account that pays 100% interest once a year, then, assuming no other deposits or withdrawals, after one year how much money would be in your account?

If $1.00 is invested in a bank account that pays 50% interest twice a year, then, assuming no other deposits or withdrawals, after one year how much money would be in your account?

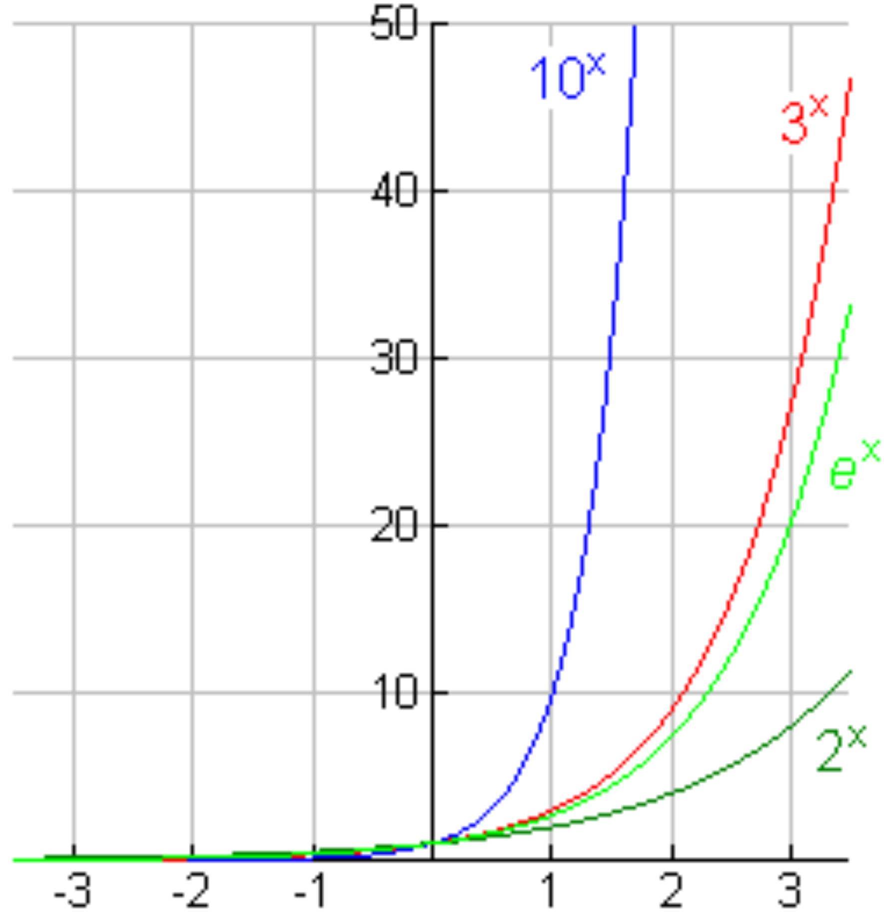
Why is the second balance larger?

The Number *e*

There are two irrational numbers (a number that has an infinite non-repeating decimal expansion) that are so important in mathematics that they have their own name, π and *e*. *e* was introduced by Euler in 1727 and has value $e \approx 2.71828\ldots.$ Base *e* is called the ***natural base*** because it makes many of the formulas of calculus much easier.

Graphs of Different Exponentials

Graph 10^x, 3^x, 2^x, and e^x, in a window $-3.5 \leq x \leq 3.5$ and $0 \leq y \leq 50$



Continuous Growth

If exponential growth is continuous then the formula becomes $Q = ae^{kt}$. If a is positive.

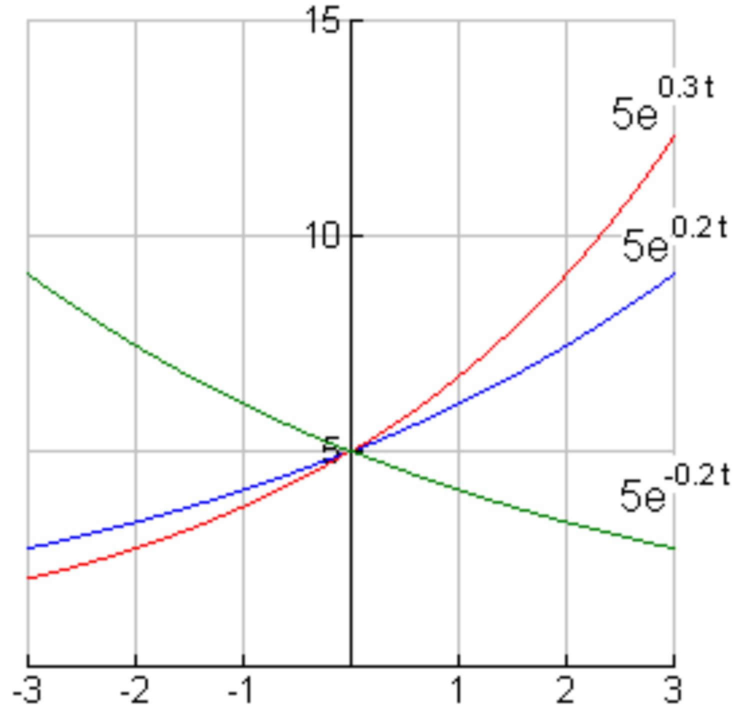
1) If $k > 0$, then Q is increasing.
2) If $k < 0$, then Q is decreasing.

Notice that $b = e^k$. So b the growth factor takes on a different form if the growth rate is continuous.

Examples of Continuous Growth Rates

Give the continuous growth rate of each of the following functions and graph each function.

1) $P = 5e^{0.2t}$
2) $Q = 5e^{0.3t}$
3) $R = 5e^{-0.2t}$



Population Example

A population increases from 7.3 million at a continuous rate of 2.2% per year.

1) Write a formula for the population.
2) Estimate graphically when the population reaches 10 million.

Caffeine Example

Caffeine leaves the body at a continuous rate of 17% per hour. How much caffeine is left 8 hours after drinking a cup of coffee containing 100 mg of caffeine?

Difference Between Annual and Continuous Growth Rates

If $P = a(1.07)^t$, with t in years, we say that P is growing at an *annual* rate of 7% per year. If $P = ae^{0.07t}$, with t in years, we say that P is growing at a *continuous* rate of 7% per year. Since $e^{0.07} = 1.0725\ldots$, we can rewrite $ae^{0.07t} \approx a(1.0725)^t$. In other words a 7% continuous growth rate corresponds to a 7.25% annual growth rate. The annual growth rate is always bigger than the corresponding continuous rate.

Connection: *e* and Compound Interest

Frequency	Balance
1 (annually)	$2.00
2 (semi-annually)	$2.25
4 (quarterly)	
12 (monthly)	
365 (daily)	
8760 (hourly)	
525,600 (each minute)	
31,536,000 (secondly)	

Connection: *e* and Compound Interest

Frequency	Balance
1 (annually)	\$2.00
2 (semi-annually)	\$2.25
4 (quarterly)	2.441406
12 (monthly)	2.613035
365 (daily)	2.714567
8760 (hourly)	2.718127
525,600 (each minute)	2.718279
31,536,000 (secondly)	2.718282

Wells Fargo Example

In November 2005, the Wells Fargo Bank offered interest at a 2.323% continuous yearly rate. Find the equivalent annual rate.

Annual vs Continuous Rates

Which is better: An account that pays 8% annual interest or an account that pays 7.5% annual interest rate compounded continuously.

The Number *e*

The number *e* = 2.71828182… is often used for the base, *b,* of the exponential function. Base *e* is called the ***natural base***.