Logarithms and Their Properties

Chapter 5

Section 1

Estimating Solutions to Exponential Equations: Table or Graphs

Consider the equation $N = 100 \cdot 2^t$, where N is the number of bacteria and t is the time, measured in 20-minute periods.

Suppose instead of using t as an input, we use N as the input. At what time t will the number of bacteria, N, equal 1000?

- Use the table function to estimate the proper value of t.
- Use the graph to estimate the value of t.

Logarithm Base 10

For any positive number x the logarithm base 10 of x is the power to which we raise 10 in order to produce x. $\log_{10} x = c$ means $10^c = x$.

Logarithms base 10 are called common logarithms and $\log_{10} x$ is also written as $\log x$.

Logs to Exponents

Rewrite the following statements using exponents instead of logs.

a) $\log 100 = 2$

b) $\log 0.01 = -2$

c) $\log 30 = 1.477121255$

Exponents to Logs

Rewrite the following statements using logs instead of exponents.

a) $10^5 = 100,000$

b) $10^{-4} = 0.0001$

c) $10^{0.8} = 6.309573445$

Logarithms Are Exponents

Logarithms are just exponents! Thinking in terms of exponents is often a good way to answer a logarithm problem. Without a calculator evaluate the following, if possible.

a) $\log 1$

b) $\log 10$

c) $\log 1{,}000{,}000$

d) $\log 0.001$

e) $\log \dfrac{1}{\sqrt{10}}$

f) $\log(-100)$

Logarithmic and Exponential Functions are Inverses

The operation of taking a logarithm "undoes" the exponential function; the logarithm and the exponential are inverse functions.

- For any N, $\log(10^N) = N$ and for $N > 0$, $10^{\log N} = N$.

Inverse Examples

Evaluate without a calculator:

a) $\log\left(10^{8.5}\right)$

b) $10^{\log 2.7}$

c) $10^{\log(x+3)}$

Summary

Properties of common logarithms:

- By definition, $y = \log x$ means $10^y = x$.
- In particular $\log 1 = 0$ and $\log 10 = 1$.
- The functions 10^x and $\log x$ are inverses, so they "undo" each other:

$$\log\left(10^x\right) = x \qquad \text{for all } x$$

$$10^{\log x} = x \qquad \text{for all } x > 0$$

Rules for Exponents

If *a* is any positive real number and *n* and *m* are any real numbers, then

$$1.\ a^n \cdot a^m = a^{(n+m)}$$

$$2.\ \frac{a^n}{a^m} = a^{(n-m)}, a \neq 0$$

$$3.\ \left(a^m\right)^n = a^{(m \cdot n)}$$

Properties for Logarithms

If *a* and *b* are any positive real numbers and *t* is any real number, then

$$1.\ \log(a \cdot b) = \log a + \log b$$

$$2.\ \log\left(\frac{a}{b}\right) = \log a - \log b$$

$$3.\ \log\left(a^t\right) = t \cdot \log a$$

Using Logarithms to Solve an Equation

Solve $100 \cdot 2^t = 337{,}000{,}000$ for t.

The Natural Logarithm

When e is used as the base for exponential functions , computations are easier with the use of another logarithm function called log base e

For any positive number x the logarithm base e of x is the power to which we raise e in order to produce x. $\ln x = c$ means $e^c = x$.

Logarithms base e are called natural logarithms and written as $\ln x$.

Some Properties of the Natural Logarithm

Properties of natural logarithms:

- By definition, $y = \ln x$ means $e^y = x$.
- In particular $\ln 1 = 0$ and $\ln e = 1$.
- The functions e^x and $\ln x$ are inverses, so they "undo" each other:

$$\ln\left(e^x\right) = x \qquad \text{for all } x$$

$$e^{\ln x} = x \qquad \text{for all } x > 0$$

Properties for Natural Logarithms

If *a* and *b* are any positive real numbers and *t* is any real number, then

$$1.\ \ln(a \cdot b) = \ln a + \ln b$$

$$2.\ \ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$$3.\ \ln\left(a^{t}\right) = t \cdot \ln a$$

Solving Using Natural Logarithms

Solve for *x*:

a) $5e^{2x} = 50$

b) $3^{x} = 100$

Misconceptions Involving Logs

$\log(a+b)$ is not the same as $\log a + \log b$

$\log(a-b)$ is not the same as $\log a - \log b$

$\log(ab)$ is not the same as $\left(\log a\right)\left(\log b\right)$

$\log\left(\dfrac{a}{b}\right)$ is not the same as $\dfrac{\log a}{\log b}$

$\log\left(\dfrac{1}{a}\right)$ is not the same as $\dfrac{1}{\log a}$

Calculator Errors Involving Logs

Consider the two expressions listed below:

$$\log 5x^2$$

$$\log \frac{17}{3}$$

Evaluate Without a Calculator

a) $\log 1$ b) $\log 0.1$ c) $\log(10)$

d) $\log\sqrt{10}$ e) $\log\left(10^5\right)$ f) $\log\left(\dfrac{1}{\sqrt{10}}\right)$

g) $10^{\log 100}$ h) $10^{\log 1}$ i) $10^{\log(0.01)}$

a) $\ln 1$ b) $\ln e^0$ c) $\ln e^5$

d) $\ln\sqrt{e}$ e) $e^{\ln 2}$ f) $e^{\ln\left(\frac{1}{\sqrt{e}}\right)}$

Solve the Equations Exactly

35. $91 = 46(1.1)^x$

39. $e^{x+4} = 10$

43. $400e^{0.1x} = 500e^{0.08x}$

Problem #34

Find a possible formula for the exponential function $S(x)$ in the figure to the right if $R(x) = 5.1403 \cdot (1.1169)^x$