Logarithms and Exponential Models

Chapter 5

Section 2

Solving Exponential Equations Using Logarithms

In the carbon-14 problem from Section 3.3, we were given the equation $Q = 200*0.886^t$, where *Q,* the amount of carbon-14 remaining is a function of the time, *t*, in thousands of years. Use logarithms to solve the equation $25 = 200*0.886^t$, for *t*.

US Population Example

The US population, *P*, in millions, is currently growing according to the formula $P = 299e^{0.009t}$, where t is in years since 2006. When is the population predicted to reach 350 million?

Another Population Example

The population of City A begins with 50,000 people and grows at 3.5% per year. The population of City B begins with a larger population of 250,000 people but grows at the slower rate of 1.6% per year. Assuming these growth rates hold constant, will the population of City A ever catch up to the population of City B? If so, when?

Doubling Time

Eventually any exponentially growing quantity doubles, or increases by 100%. Since its percent growth rate is constant, the time it takes for the quantity to grow by 100% is also constant. This time period is called the *doubling time*.

Doubling Time Example - Turtles

In an earlier example we used the equation $P = 175*1.145^t$ to describe the turtle population as a function of time t.

a) Find the time needed for the turtle population to double.

b) How long does it take for this population to quadruple its initial size?

c) How long to increase its size by a factor of 8?

Another Doubling Time Question

A population doubles in size every 20 years. What is its continuous growth rate?

Half-Life

Just as an exponentially growing quantity doubles in a fixed amount of time, an exponentially decaying quantity decreases by a factor of 2 in a fixed amount of time, called the *half-life* of the quantity.

Computing the Half-Life of Carbon-14

Carbon-14 decays radioactively at a constant annual rate of 0.0121%. Show that the half-life of carbon-14 is about 5728 years.

Computing Decay Rate from Half-Life

The quantity, Q, of a substance decays according to the formula $Q = Q_0e^{-kt}$, where t is in minutes. The half-life of the substance is 11 minutes. What is the value of k?

Converting Between Exponential Forms $Q = ab^t$ and $Q = ae^{kt}$

Any exponential function can be written in either of the two forms $Q = ab^t$ or $Q = ae^{kt}$. If $b = e^k$, so $k = \ln b$, the two formulas represent the same function.

Conversion Examples

1) Convert the exponential function $P = 175*1.145^t$ to the form $P = ae^{kt}$.

2) Convert the formula $Q = 7e^{0.3t}$ to the form $Q = ab^t$.

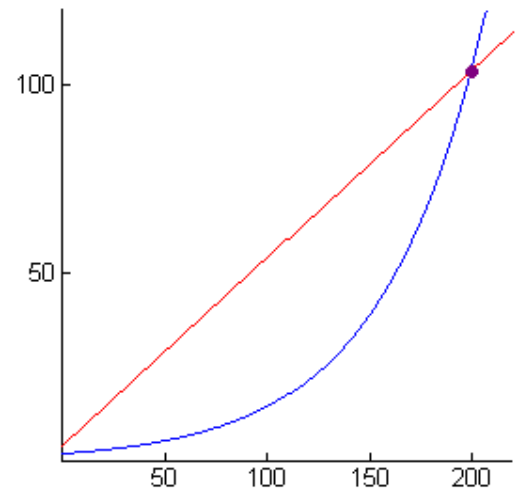
3) Assuming t is in years, find the continuous and annual percent growth rates in the above two examples

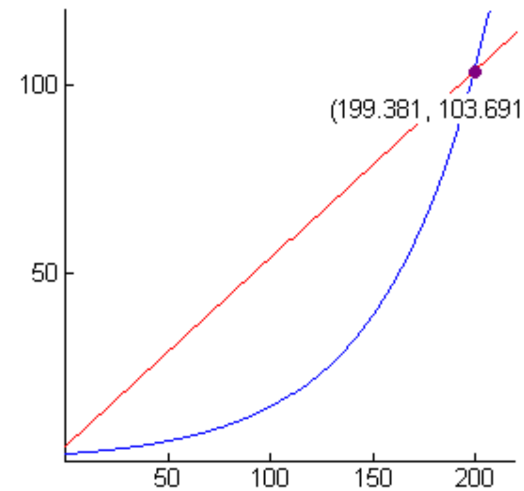
Annual to Continuous Conversion

Find the continuous percent growth rate of $Q = 200*0.886^t$, where t is in thousands of years.

Exponential Growth Problems That Cannot Be Solved By Logarithms

With t in years, the population of a country (in millions) is given by $P = 2(1.02)^t$, while the food supply (in millions of people that can be fed) is given by $N = 4 + 0.5t$. Determine the year in which the country first experiences food shortages.





Exercises #13 and #14

Give the starting value *a*, the growth rate *r*, and the continuous growth rate *k*.

#13 $Q = 230(1.182)^t$

#15 $Q = 0.81(2)^t$

Problem #37

Total power generated by wind worldwide doubles every 3 years. In 2008, world wind-energy generating capacity was about 90 thousand megawatts. Find the continuous growth rate and give a formula for wind generating capacity W (in thousand megawatts) as a function of t, number of years since 2008