

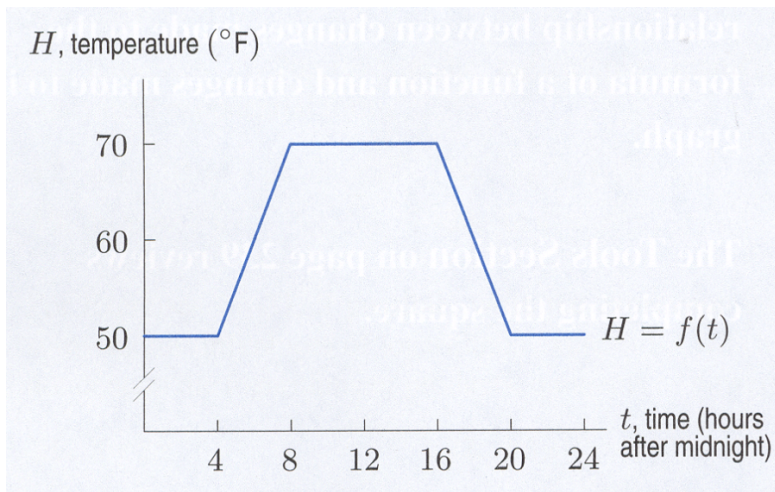
Vertical and Horizontal Shifts

Chapter 6

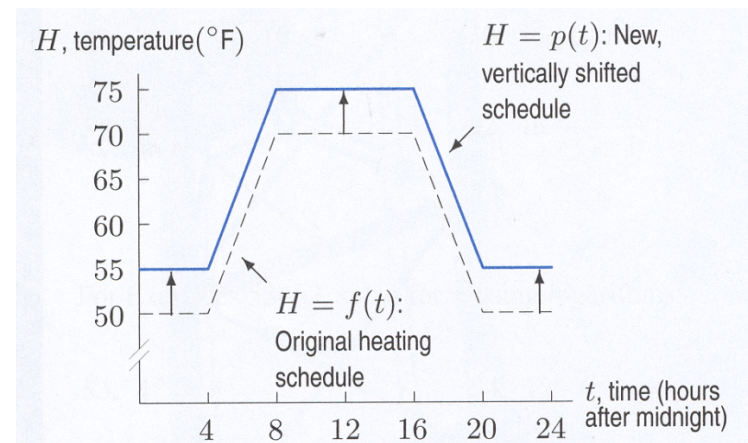
Section 1

The Heating Schedule for an Office Building

To save money, an office building is kept warm only during business hours. The graph below shows the temperature, H , in $^{\circ}\text{F}$, as a function of time, t , in hours after midnight.



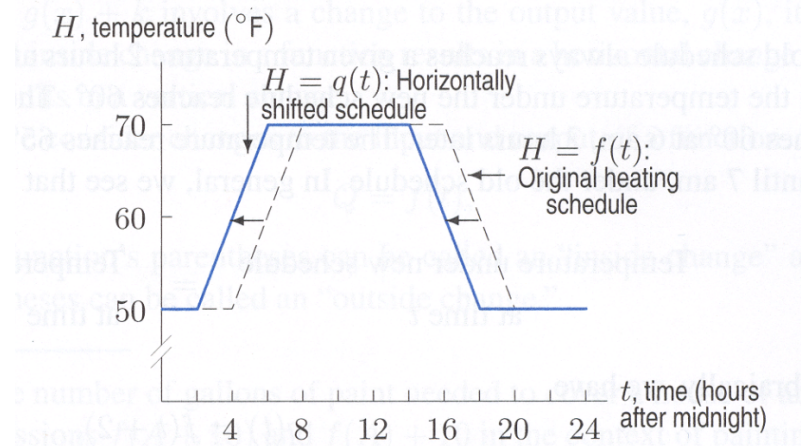
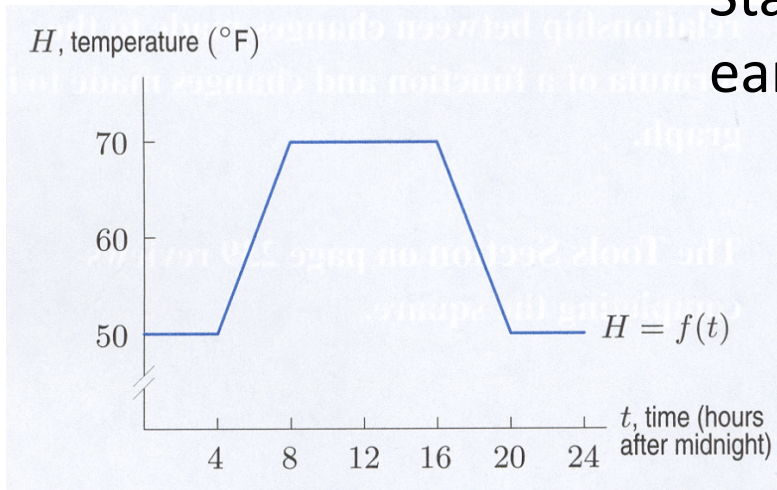
Raise the building temperature 5 $^{\circ}\text{F}$.



The Heating Schedule for an Office Building

To superintendent then changes the original heating schedule to start two hours earlier. Explain how this schedule is different from the original one.

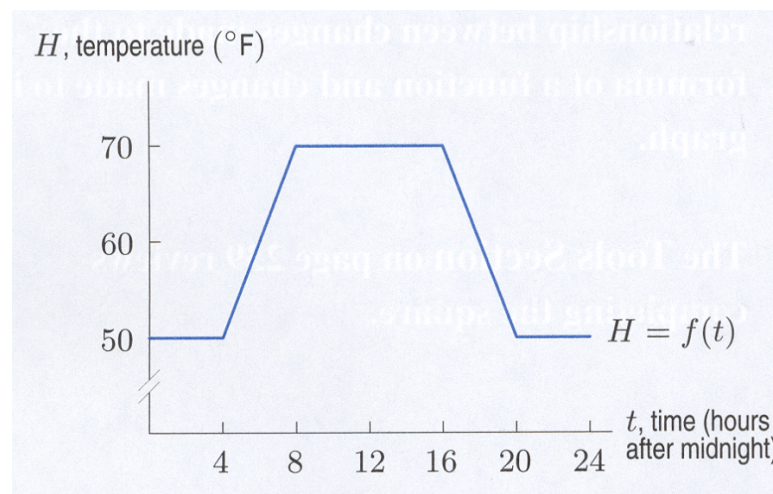
Start the building warming 2 hours earlier.



Formula for a Vertical Shift

Let $H = f(t)$ be the original heating schedule.
Let $H = p(t)$ be the new warmer heating schedule. How are the formulas for $f(t)$ and $p(t)$ related?

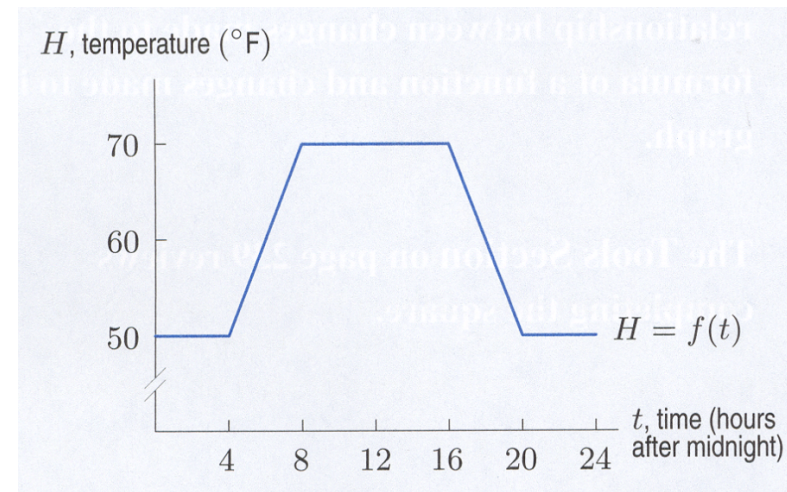
Use the graph of $f(t)$ to find the value $p(6)$.



Formula for a Horizontal Shift

In the second heating example, the heating schedule was changed to 2 hours earlier, shifting the graph horizontally 2 units to the left. Find a formula for q , the new schedule, in terms of the original schedule

Use the formula for $q(t)$ and the graph to the right to calculate $q(14)$



Summary of Vertical and Horizontal Shifts

If $y = g(x)$ is a function and k is a constant, then the graph of

- $y = g(x) + k$ is the graph of $y = g(x)$ shifted vertically $|k|$ units. If k is positive, the shift is up; if k is negative, the shift is down.
- $y = g(x + k)$ is the graph of $y = g(x)$ shifted horizontally $|k|$ units. If k is positive, the shift is to the left; if k is negative, the shift is to the right.

Inside and Outside Changes

Since $y = g(x + k)$ involves a change to the input value, x , it is called an ***inside change*** to g . Similarly, since $y = g(x) + k$ involves a change to the output value, $g(x)$, it is called an ***outside change*** to g . In general, an inside change in a function results in a horizontal change in its graph, whereas an outside change results in a vertical change.

House Painting Example

If $n = f(A)$ gives the number of gallons of paint needed to cover a house of area A ft², explain the meaning of the expressions $f(A + 10)$ and $f(A) + 10$ in the context of painting.

Average Weight

Let $s(t)$ be the average weight (in pounds) of a baby at age t months. The weight, V , of a particular baby named Jonah is related to the average weight function $s(t)$ by the equation $V = s(t) + 2$. Find Jonah's weight at ages $t = 3$ and $t = 6$ months. What can you say about Jonah's weight in general?

Average Weight of Ben

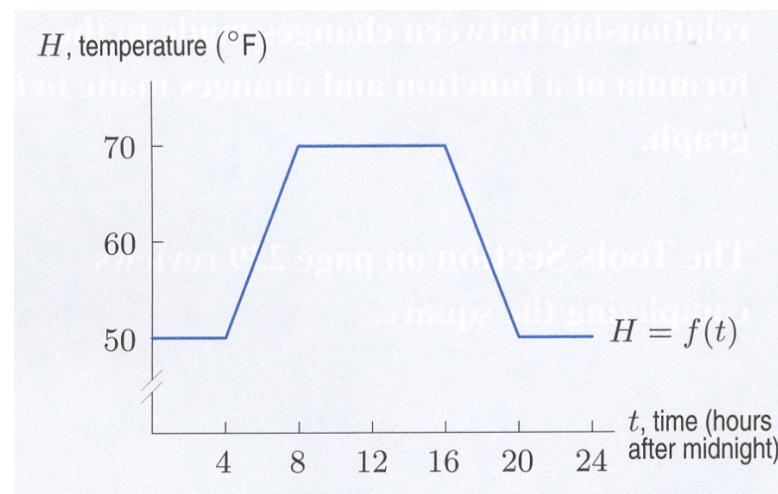
The weight, W , of another baby named Ben is related to $s(t)$ by the equation $W = s(t + 4)$.

What can you say about Ben's weight at age $t = 3$ months? At $t = 6$ months? Assuming that babies increase in weight over the first year of life, decide if Ben is of average weight for his age, above average, or below average.

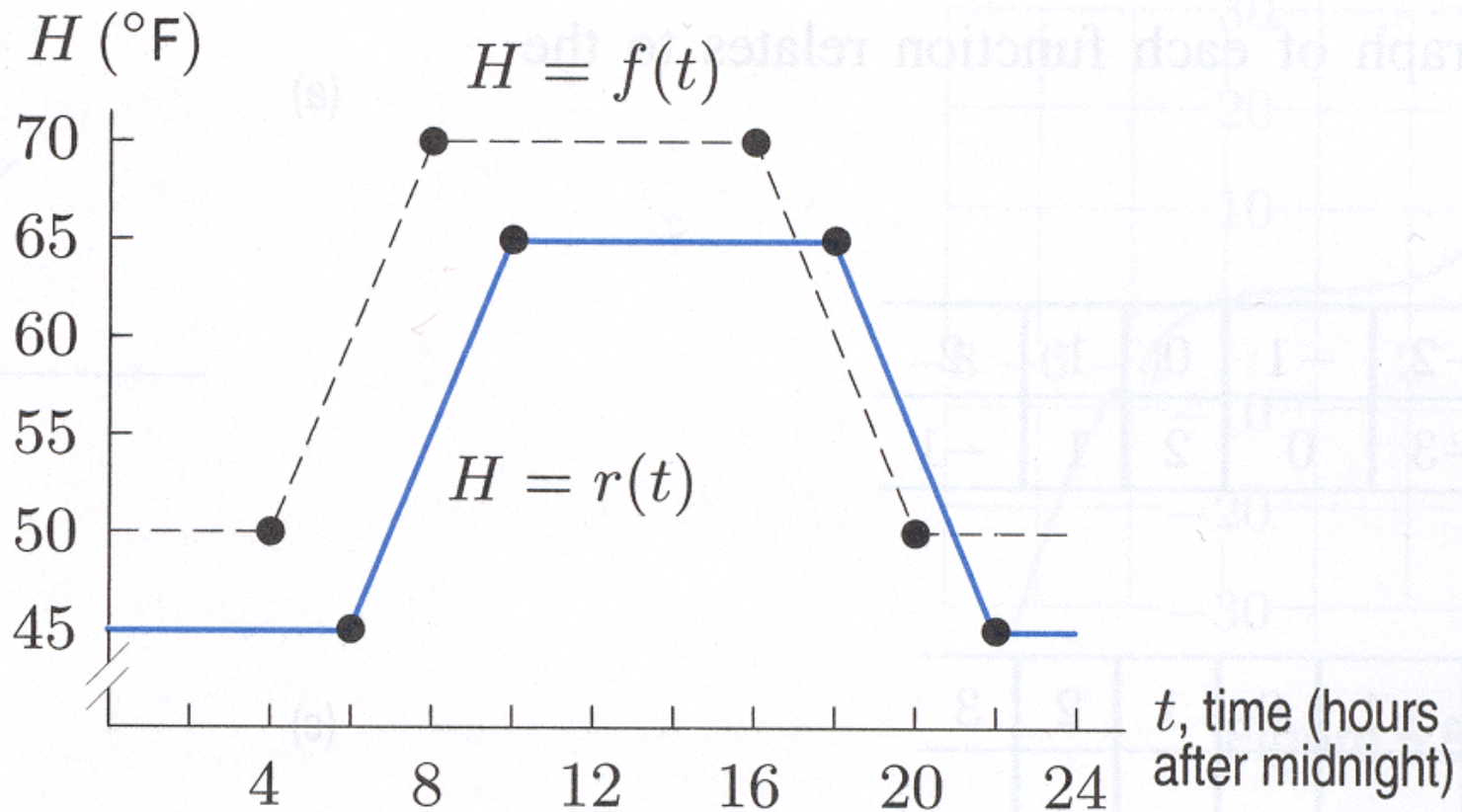
Combining Vertical and Horizontal Shifts

Let r be the transformation of the heating schedule function, $H = f(t)$, defined by the equation $r(t) = f(t - 2) - 5$.

1. Sketch the graph of $H = r(t)$.
2. Describe in words the heating schedule determined by r .



Solution for Combined Shifts



Shifting Graphs

A graph of $f(x) = x^2$ is pictured to the left below. Define g by shifting the graph to the right 2 units and down 1 unit; see the figure to the right. Find a formula for g in terms of f . Find a formula for g in terms of x .

