Introduction to Periodic Functions

Chapter 7

Section 1

The World's Largest Ferris Wheel

To celebrate the millennium, British Airways funded construction of the "London Eye," the worlds largest Ferris wheel. The wheel is located on the south bank of the river Thames, in London, England, measures 450 feet in diameter, and carries up to 800 passengers in 32 capsules. It turns continuously, completing a single rotation once every 30 minutes. This is slow enough for people to hop on and off while it turns.

London Eye



Ferris Wheel Height as a Function of Time

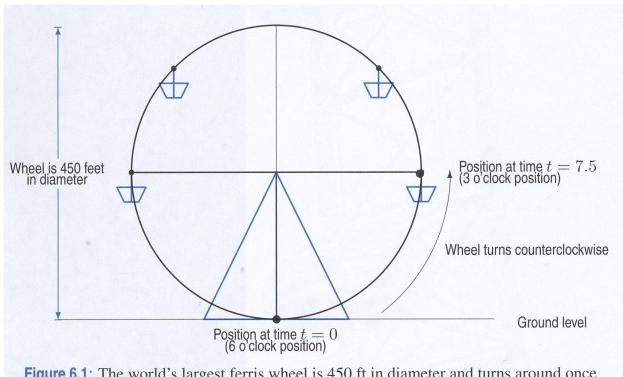
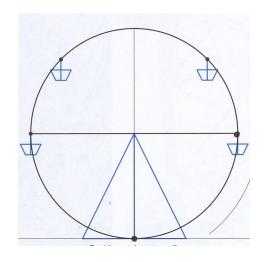


Figure 6.1: The world's largest ferris wheel is 450 ft in diameter and turns around once every 30 minutes. Seats not drawn to scale

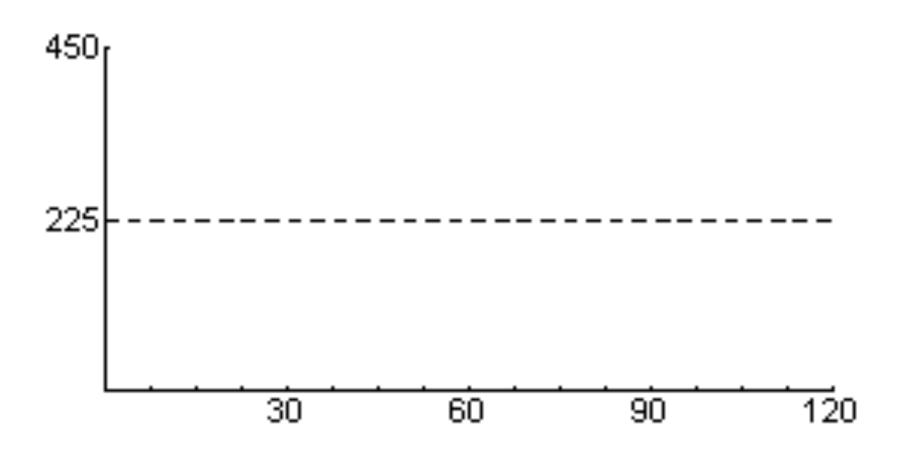
| | <i>l- (+</i>) | | | |
|---------------|-----------------|--|--|--|
| $\mid t \mid$ | h(t) | | | |
| (min) | (feet) | | | |
| 0 | | | | |
| 7.5 | | | | |
| 15 | | | | |
| 22.5 | | | | |
| 30 | | | | |

Multiple Times Around the Wheel

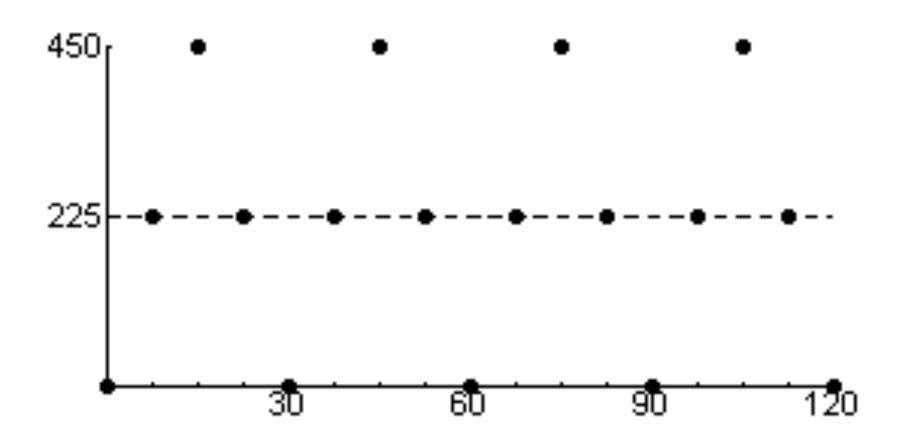
| t (min) | 0 | 7.5 | 15 | 22.5 | 30 | 37.5 | 45 | 52.5 | |
|---------------------------------|----|------|----|------|----|------|-----|-------|-----|
| <i>f</i> (<i>t</i>) (feet) | | | | | | | | | |
| t (min) | 60 | 67.5 | 75 | 82.5 | 90 | 97.5 | 105 | 112.5 | 120 |
| <i>f</i> (<i>t</i>) (feet) | | | | | | | | | |



Graphing the Ferris Wheel Function



Graphing the Ferris Wheel Function



Filing in the Gap of the Ferris Wheel Function

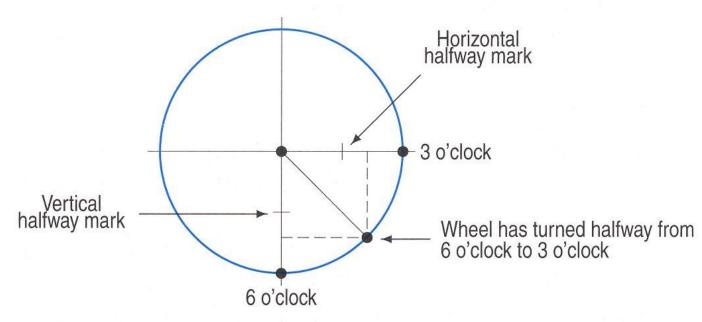


Figure 6.5: As the wheel turns half the way from 6 o'clock to 3 o'clock, the seat rises less than half the vertical distance but glides more than half the horizontal distance

Plot the Data Points

| t | 0 | 3.75 | 7.5 | 11.25 | 15 | 18.75 | 22.5 | 26.25 | 30 |
|------|---|------|-----|-------|-----|-------|------|-------|----|
| f(t) | 0 | 65.9 | 225 | 384.1 | 450 | 384.1 | 225 | 65.9 | 0 |

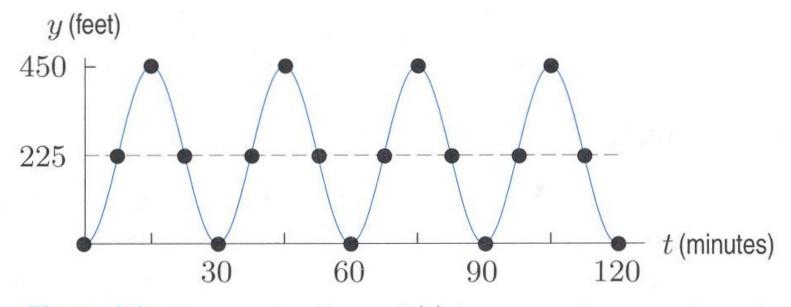


Figure 6.6: The graph of y = f(t) is a smooth wave-shaped curve

Periodic Functions: Period

The ferris wheel function, f, is said to be **periodic**. The smallest time interval during which a function completes one full cycle is called its *period* and is represented as a horizontal distance, i.e. a distance on the x-axis.

One way to think about the period is in terms of horizontal shifts, f(t + 30) = f(t).

Periodic Functions: Midline and Amplitude

The *midline* of a periodic function is the horizontal line midway between the function's maximum and minimum values. The *amplitude* is the vertical distance between the function's maximum (or minimum) value and the midline.

Periodic Functions: Period, Midline, and Amplitude

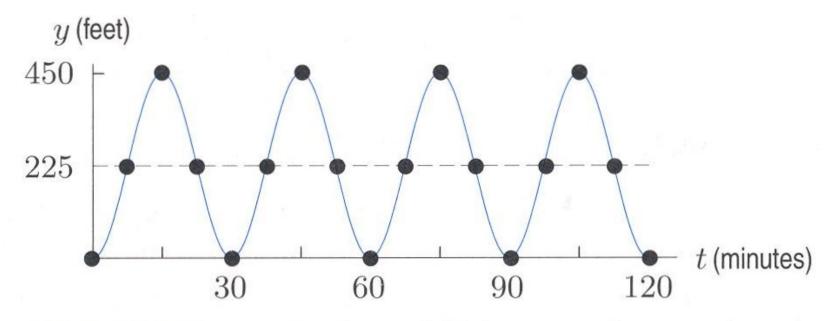


Figure 6.6: The graph of y = f(t) is a smooth wave-shaped curve