The Sine and Cosine Functions

Chapter 7
Section 2
Using Angles to Measure Position on a Circle

Measure angles with respect to the horizontal, not the vertical so that 0° describes the 3 o’clock position.

Positive angles are measured in the counterclockwise direction, negative angles in the clockwise direction.

Large angles (greater than 360° or less than -360°) wrap around a circle more than once.
Example 1

Sketch angles showing the following positions on the Ferris wheel:  a) 90°  b) -90°  c) 720°
# Height on the Ferris Wheel as a Function of Angle

<table>
<thead>
<tr>
<th>$\theta$ (degrees)</th>
<th>-90°</th>
<th>0°</th>
<th>90°</th>
<th>180°</th>
<th>270°</th>
<th>360°</th>
<th>450°</th>
<th>540°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ (feet)</td>
<td>0</td>
<td>225</td>
<td>450</td>
<td>225</td>
<td>0</td>
<td>225</td>
<td>450</td>
<td>225</td>
</tr>
<tr>
<td>$\theta$ (degrees)</td>
<td>630°</td>
<td>720°</td>
<td>810°</td>
<td>900°</td>
<td>990°</td>
<td>1080°</td>
<td>1170°</td>
<td>1280°</td>
</tr>
<tr>
<td>$y$ (feet)</td>
<td>0</td>
<td>225</td>
<td>450</td>
<td>225</td>
<td>0</td>
<td>225</td>
<td>450</td>
<td>225</td>
</tr>
</tbody>
</table>
Example Points on the Unit Circle

Use the unit circle shown to the right to find the point on the circle corresponding to the angles 90°, 180°, and 210°.
Definition of Sine and Cosine

Suppose $P = (x, y)$ is the point on the unit circle specified by the angle $\theta$. We define the functions, cosine of $\theta$, or $\cos \theta$, and the sine of $\theta$, or $\sin \theta$, by the formulas $\cos \theta = x$ and $\sin \theta = y$.

In other words $\cos \theta$ is the $x$-coordinate of the point on the unit circle specified by the angle $\theta$ and $\sin \theta$ is the $y$-coordinate.
Definition of Sine and Cosine
Computations – Example 2

Find the values of $\sin t$ and $\cos t$

- $0^\circ$
- $90^\circ$
- $180^\circ$
- $270^\circ$
Computation – Example 3

In the figure to the right, find the coordinates of the point Q on the unit circle.

Find the lengths of the line segments labeled $m$ and $n$. 
The Sine and Cosine Functions in Right Triangles

If $\theta$ is an angle in a right triangle (other than the right angle),

\[
\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}.
\]

**Figure 7.20:** A right triangle shown with the unit circle

**Figure 7.21:** A triangle similar to the triangle in Figure 7.20
Example 4

Referring to the figure to the right, find $\sin \theta$ and $\cos \theta$.

Figure 7.22: Find $\sin \theta$ and $\cos \theta$
Problem #13

Find an angle $\phi$, with $0^\circ < \phi < 360^\circ$, that has the same

a) Cosine as 53°
b) Sine as 53°
Problem #15

For the angle $\phi$ shown in the figure to the right sketch each of the following angles

a) $180 + \phi$
b) $180 - \phi$
c) $90 - \phi$
d) $360 - \phi$
Problem #23

A ladder 3 meters long leans against a house, making an angle $\alpha$ with the ground. How far is the base of the ladder from the base of the wall, in terms of $\alpha$? Include a sketch.