The Sine and Cosine Functions

Chapter 7

Section 2

Using Angles to Measure Position on a Circle

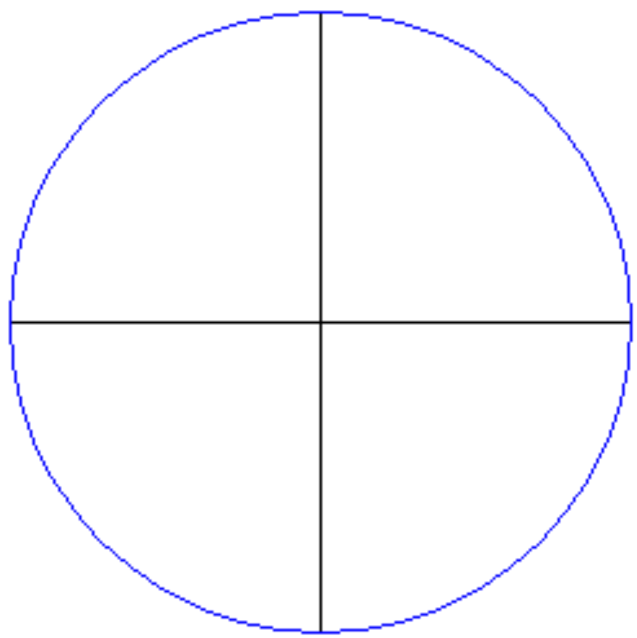
Measure angles with respect to the horizontal, not the vertical so that 0° describes the 3 o'clock position.

Positive angles are measured in the counterclockwise direction, negative angles in the clockwise direction.

Large angles (greater than 360° or less than -360°) wrap around a circle more than once.

Example 1

Sketch angles showing the following positions on the Ferris wheel: a) 90° b) -90° c) 720°

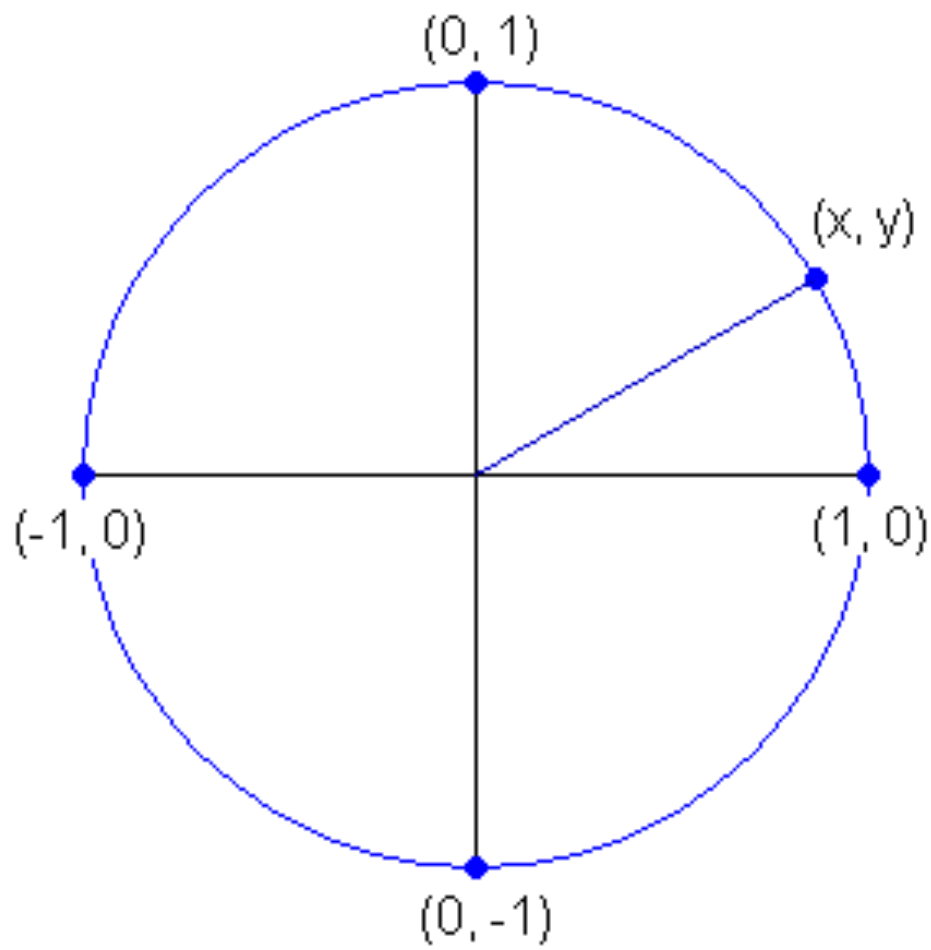


Height on the Ferris Wheel as a Function of Angle

θ (degrees)	-90°	0°	90°	180°	270°	360°	450°	540°
y (feet)	0	225	450	225	0	225	450	225
θ (degrees)	630°	720°	810°	900°	990°	1080°	1170°	1280°
y (feet)	0	225	450	225	0	225	450	225

Example Points on the Unit Circle

Use the unit circle shown to the right to find the point on the circle corresponding to the angles 90°, 180°, and 210°.

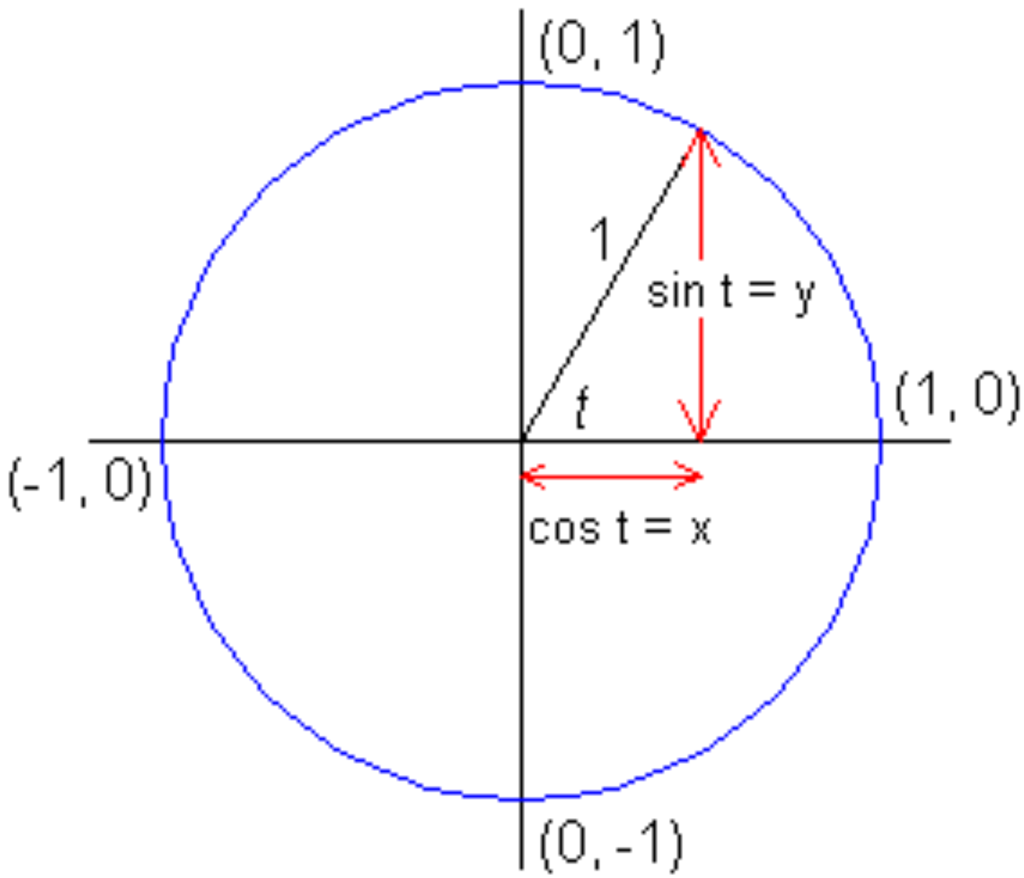


Definition of Sine and Cosine

Suppose $P = (x, y)$ is the point on the unit circle specified by the angle θ. We define the functions, **cosine** of θ, or cos θ, and the **sine** of θ, or sin θ, by the formulas $\cos\theta = x$ and $\sin\theta = y$.

In other words cos θ is the *x*-coordinate of the point on the unit circle specified by the angle θ and sin θ is the *y*-coordinate.

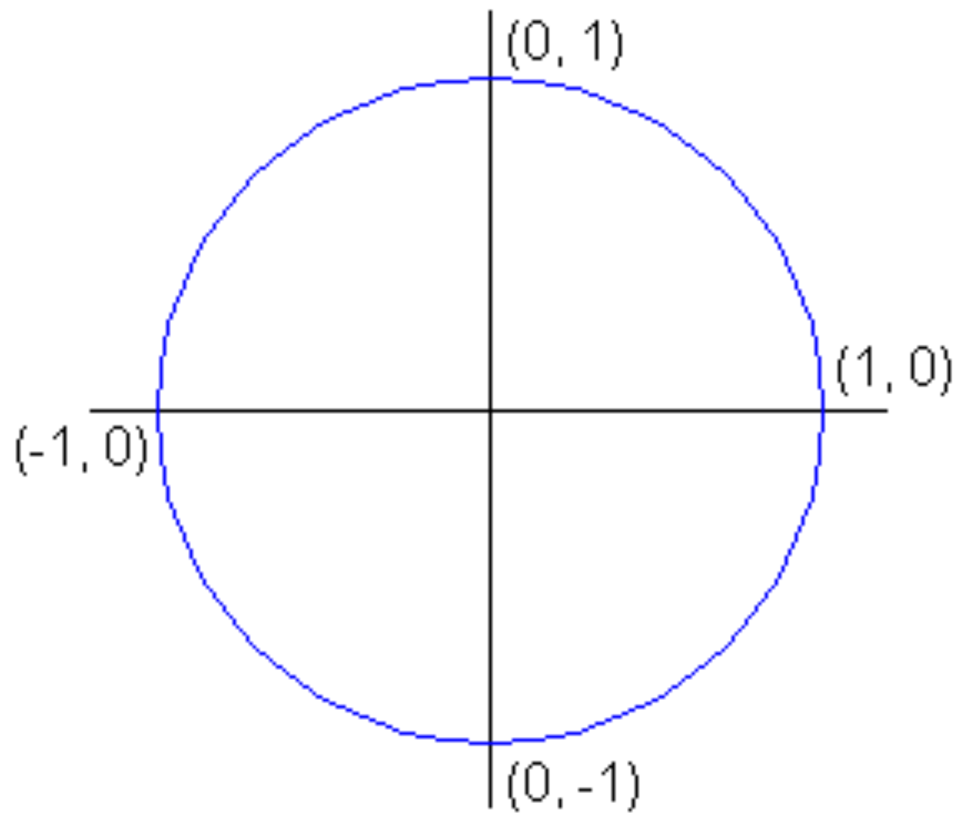
Definition of Sine and Cosine



Computations – Example 2

Find the values of sin t and cos t

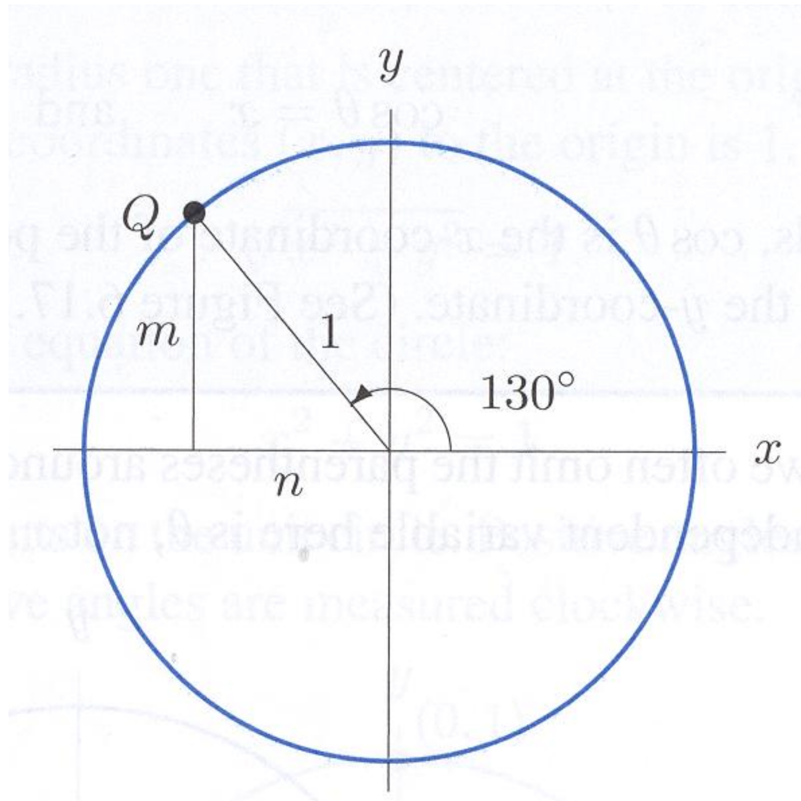
- 0°
- 90°
- 180°
- 270°



Computations – Example 3

In the figure to the right, find the coordinates of the point *Q* on the unit circle.

Find the lengths of the line segments labeled *m* and *n*.



The Sine and Cosine Functions in Right Triangles

If θ is an angle in a right triangle (other than the right angle),

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}, \qquad \cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}.$$

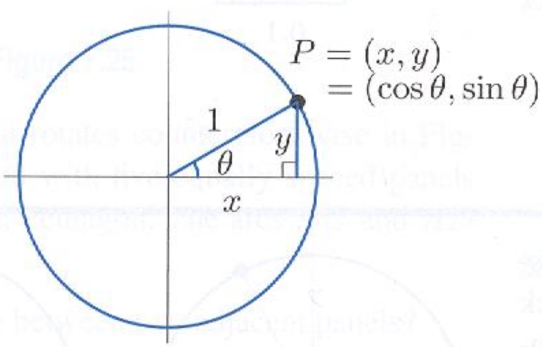


Figure 7.20: A right triangle shown with the unit circle

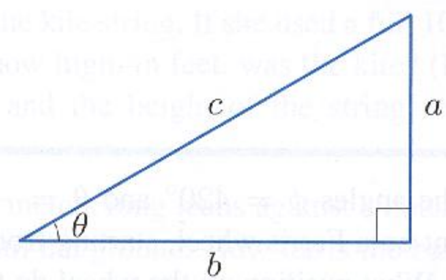


Figure 7.21: A triangle similar to the triangle in Figure 7.20

Example 4

Referring to the figure to the right, find sin θ and cos θ.

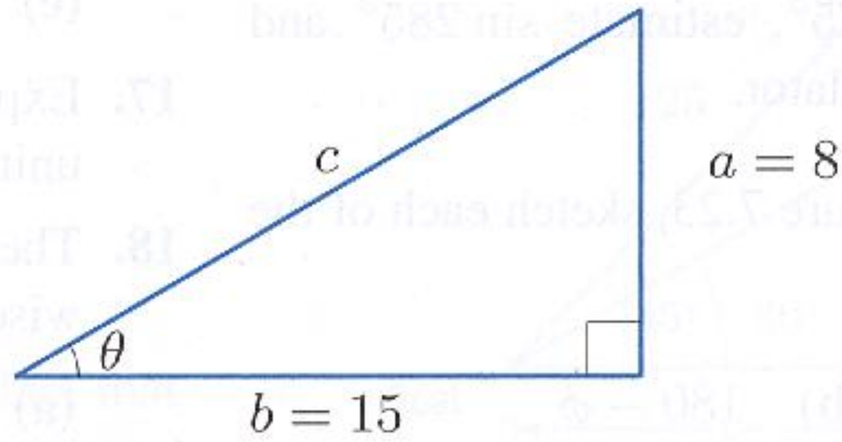


Figure 7.22: Find $\sin\theta$ and $\cos\theta$

Problem #13

Find an angle ϕ, with $0° < \phi < 360°$, that has the same

a) Cosine as 53°

b) Sine as 53°

Problem #15

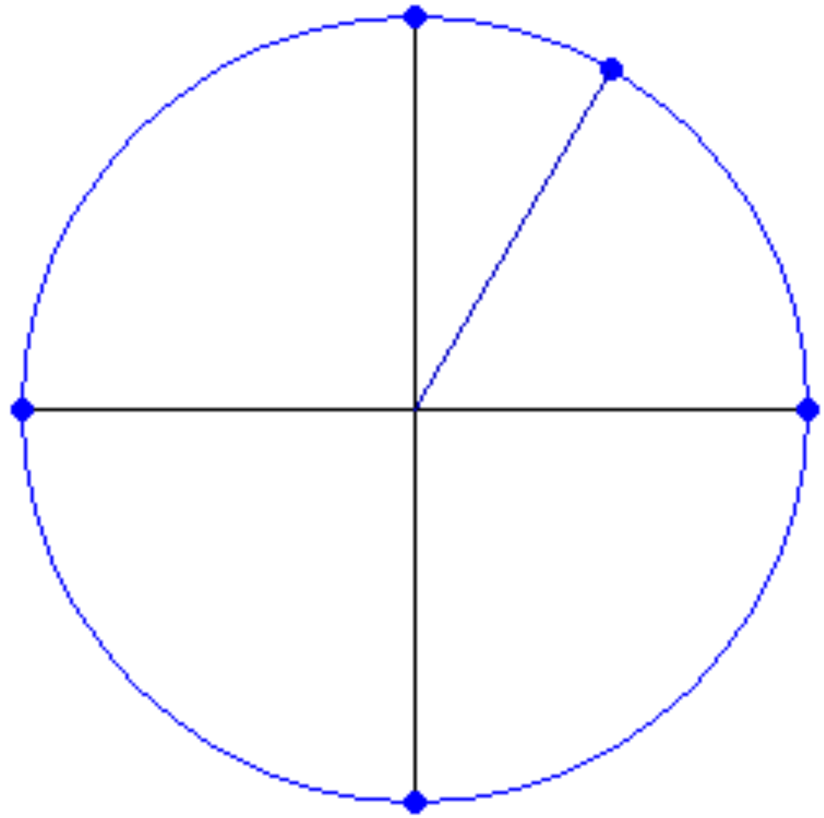
For the angle ϕ shown in the figure to the right sketch each of the following angles

a) $180 + \phi$

b) $180 - \phi$

c) $90 - \phi$

d) $360 - \phi$



Problem #23

A ladder 3 meters long leans against a house, making an angle α with the ground. How far is the base of the ladder from the base of the wall, in terms of α? Include a sketch.