Graphs of the Sine and Cosine

Chapter 7
Section 3
Exact Values of the Sine and Cosine

Previously we calculated that the following values:

\[
\cos 90^\circ = 0, \quad \sin 90^\circ = 1, \quad \cos 180^\circ = -1, \quad \sin 180^\circ = 0
\]

Evaluate \( \sin \theta \) and \( \cos \theta \) for \( \theta = 0^\circ, 270^\circ, \) and \( 360^\circ \).
Exact Values for Angles, $30^\circ$, $45^\circ$, $60^\circ$
Graphs of the Sine and Cosine Functions

Table 7.4  Values of $\sin \theta$ and $\cos \theta$ for $0 \leq \theta < 360^\circ$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\cos \theta$</th>
<th>$\sin \theta$</th>
<th>$\theta$</th>
<th>$\cos \theta$</th>
<th>$\sin \theta$</th>
<th>$\theta$</th>
<th>$\cos \theta$</th>
<th>$\sin \theta$</th>
<th>$\theta$</th>
<th>$\cos \theta$</th>
<th>$\sin \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>1</td>
<td>180</td>
<td>-1</td>
<td>0</td>
<td>270</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>30</td>
<td>0.87</td>
<td>0.5</td>
<td>120</td>
<td>-0.5</td>
<td>0.87</td>
<td>210</td>
<td>-0.87</td>
<td>-0.5</td>
<td>300</td>
<td>0.5</td>
<td>-0.87</td>
</tr>
<tr>
<td>45</td>
<td>0.71</td>
<td>0.71</td>
<td>135</td>
<td>-0.71</td>
<td>0.71</td>
<td>225</td>
<td>-0.71</td>
<td>-0.71</td>
<td>315</td>
<td>0.71</td>
<td>-0.71</td>
</tr>
<tr>
<td>60</td>
<td>0.5</td>
<td>0.87</td>
<td>150</td>
<td>-0.87</td>
<td>0.5</td>
<td>240</td>
<td>-0.5</td>
<td>-0.87</td>
<td>330</td>
<td>0.87</td>
<td>-0.5</td>
</tr>
</tbody>
</table>
Identify Properties of Sine and Cosine

Range, Period
Amplitude

Compare the graph of \( y = \sin t \) to the graphs of \( y = 2\sin t \) and \( y = -0.5\sin t \) for \( 0 \leq t \leq 2\pi \). How are the graphs similar? Different? Amplitudes?
Midline

Consider the graph \( y = \cos t + 2 \).

Notice the midline is also shifted 2 units up.

**Figure 6.43:** The graph of \( y = \cos t + 2 \) and its midline \( y = 2 \)
Graph the ferris wheel function giving your height, $h = f(\theta)$, in feet, above the ground as a function of the angle $\theta$.

**Figure 6.44**: On the ferris wheel: Height, $h$, above ground as a function of the angle, $\theta$
Coordinates of a Point on a Circle of Radius $r$

The coordinates $(x, y)$ of the point $Q$ are given by $x = r \cos \theta$ and $y = r \sin \theta$. 

\[
\frac{x}{\cos \theta} = \frac{r}{1} \quad \text{and} \quad \frac{y}{\sin \theta} = \frac{r}{1}
\]
Calculations on a Circle of Radius 5

Find the coordinates of the points A, B, and C in the figure pictured to the right accurate to three decimal places.
Another Example Using Radius 5

In the figure to the right, write the height of the point $P$ above the $x$-axis as a function of the angle $\theta$. Also graph this function.
Height on the Ferris Wheel as a Function of Angle

Find your height above the ground for both of the angles $\theta$ pictured above.
Heights on the London Eye

The London Eye has a radius of 225 feet. Find your height above the ground as a function of the angle $\theta$ measured from the 3 o’clock position.
Graph the Ferris wheel function, found previously, giving your height, $h = f(\theta)$, in feet, above ground as a function of the angle $\theta$. What are the period, midline, and amplitude?
Exercise #3

Find the midline and amplitude of the periodic function pictured to the right.
Exercises #11, #13, and #17

Find the coordinates of the point at the given angle on a circle of radius 3.8 centered at the origin.

11. -270°
13. 1426°
17. 225°
Problem #27

Estimate the period, midline, and amplitude of the periodic function pictured to the right.
Problem #34

A Ferris wheel is 20 meters in diameter and makes one revolution every 4 minutes. For how many minutes of any revolution will your seat be above 15 meters?