Graphs of the Sine and Cosine

Chapter 7

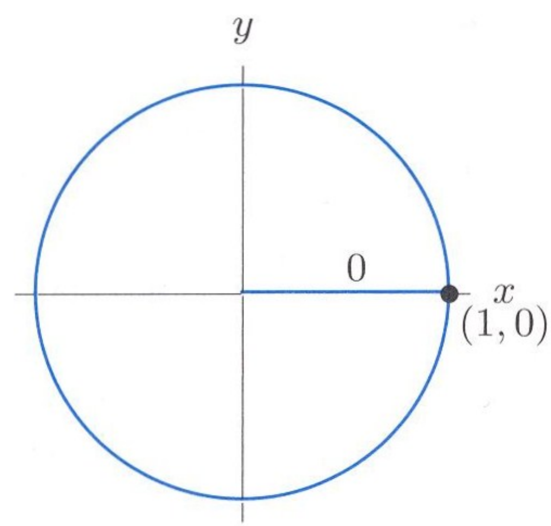
Section 3

Exact Values of the Sine and Cosine

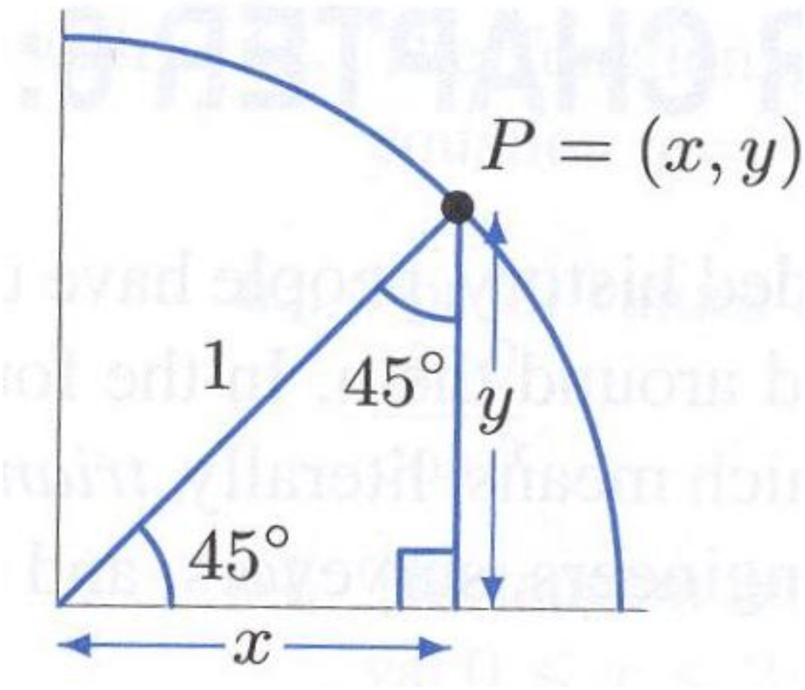
Previously we calculated that the following values:

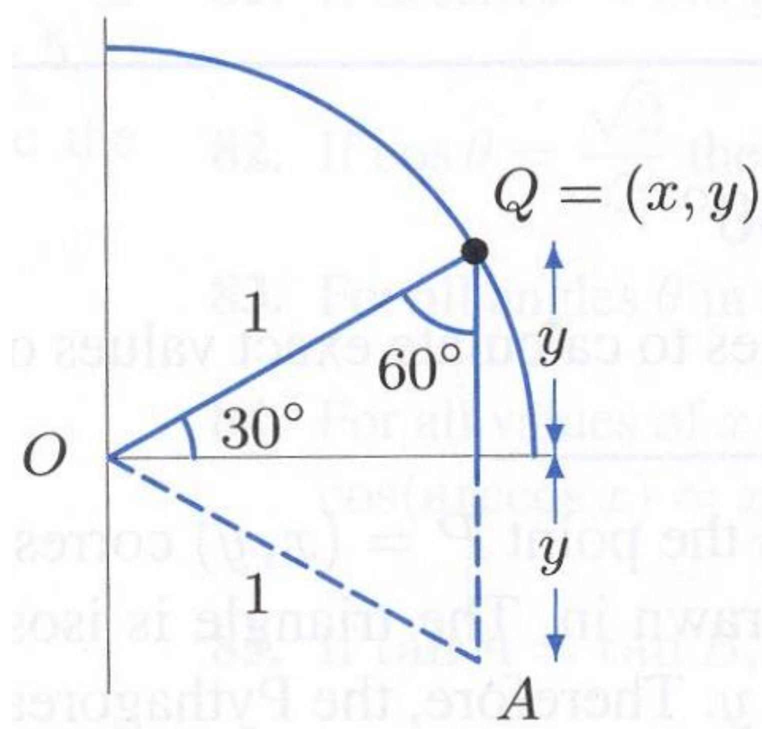
$$\cos 90^\circ = 0, \quad \sin 90^\circ = 1, \quad \cos 180^\circ = -1, \quad \sin 180^\circ = 0$$

Evaluate $\sin\theta$ and $\cos\theta$ for θ = 0°, 270°, and 360°.



Exact Values for Angles, 30°, 45°, 60°

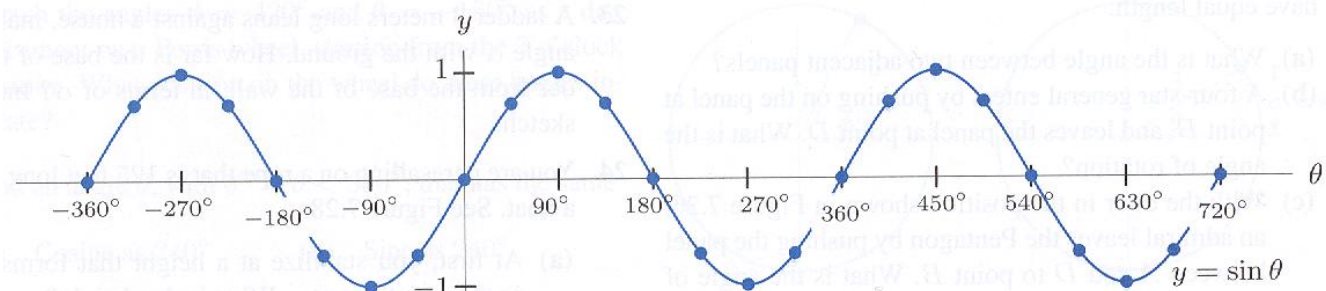


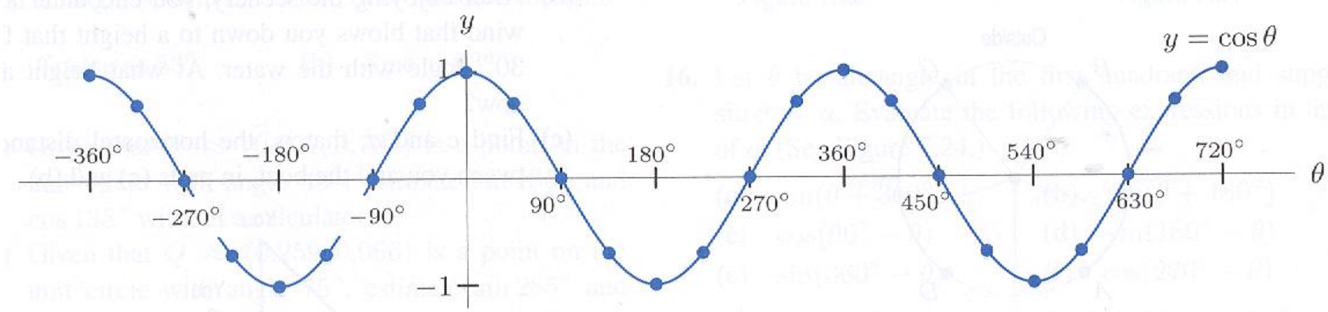


Graphs of the Sine and Cosine Functions

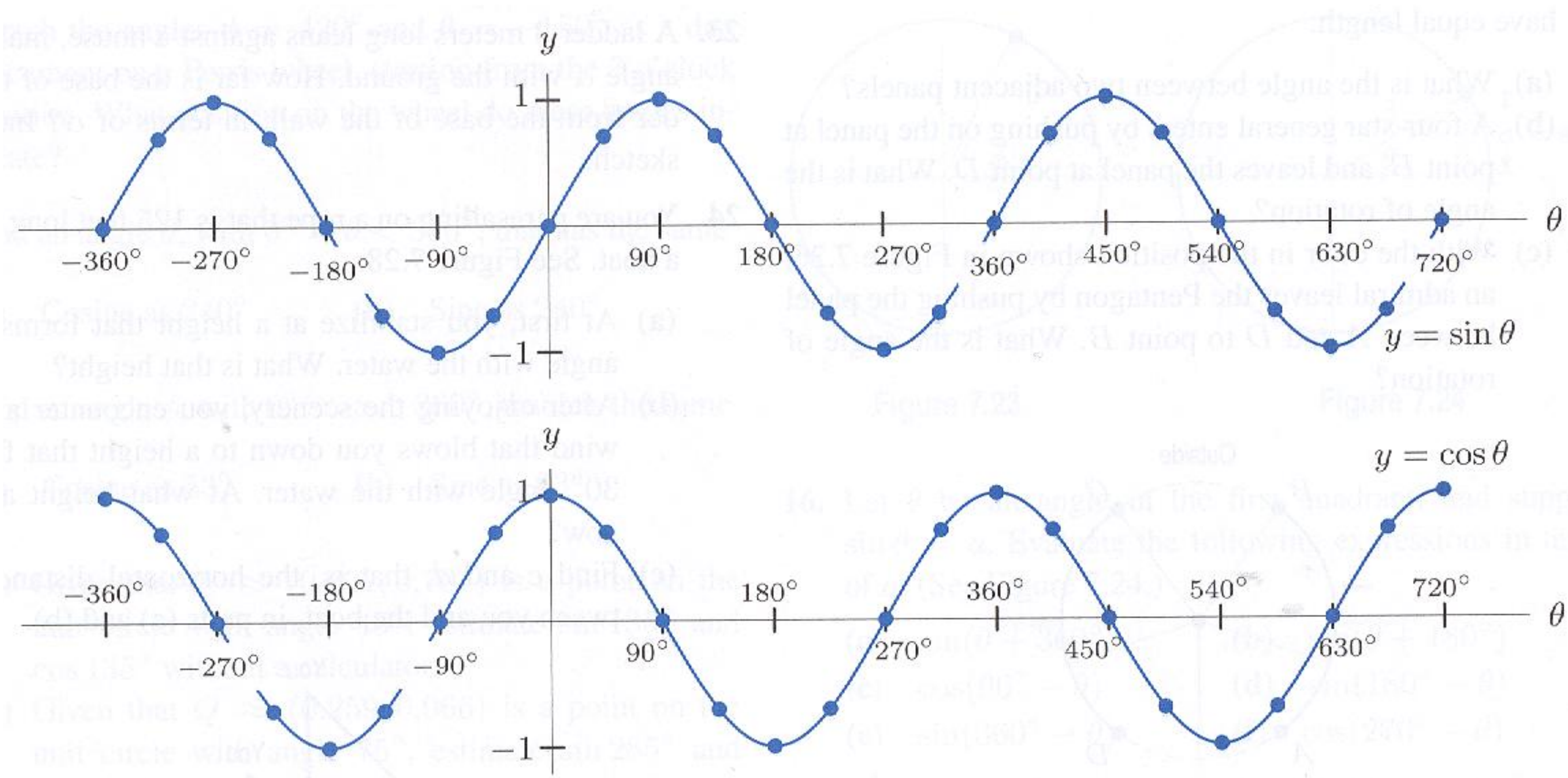
Table 7.4 *Values of* $\sin\theta$ *and* $\cos\theta$ *for* $0 \le \theta < 360°$

θ	$\cos\theta$	$\sin\theta$	θ	$\cos\theta$	$\sin\theta$	θ	$\cos\theta$	$\sin\theta$	θ	$\cos\theta$	$\sin\theta$
0	1	0	90	0	1	180	−1	0	270	0	−1
30	0.87	0.5	120	−0.5	0.87	210	−0.87	−0.5	300	0.5	−0.87
45	0.71	0.71	135	−0.71	0.71	225	−0.71	−0.71	315	0.71	−0.71
60	0.5	0.87	150	−0.87	0.5	240	−0.5	−0.87	330	0.87	−0.5





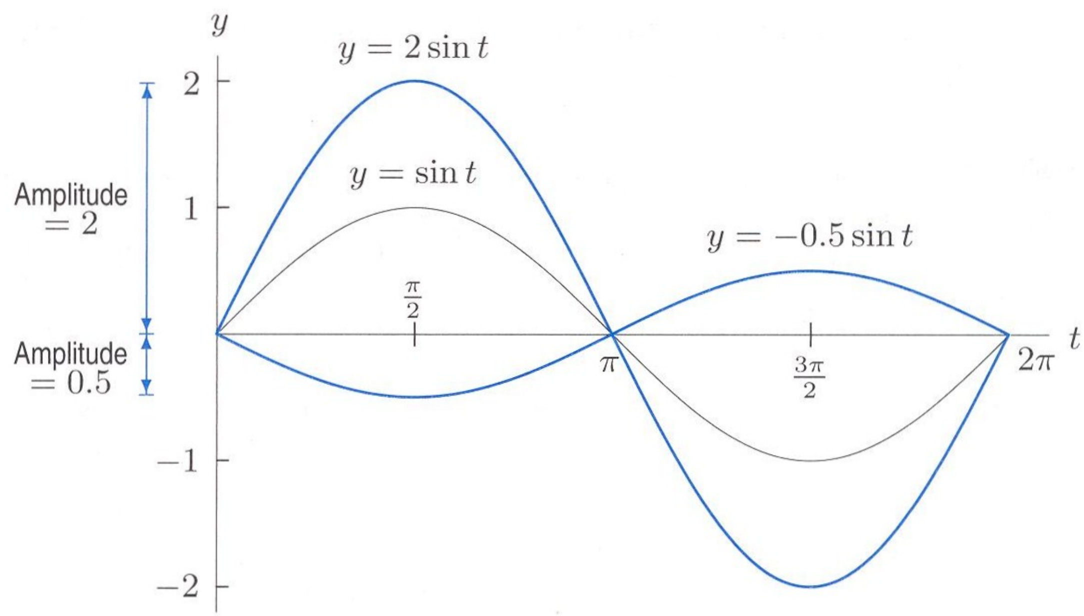
Identify Properties of Sine and Cosine



Range, Period

Amplitude

Compare the graph of y = sin t to the graphs of y = 2sin t and y = -0.5sin t for $0 \le t \le 2\pi$. How are the graphs similar? Different? Amplitudes?



Midline

Consider the graph $y = \cos t + 2$.

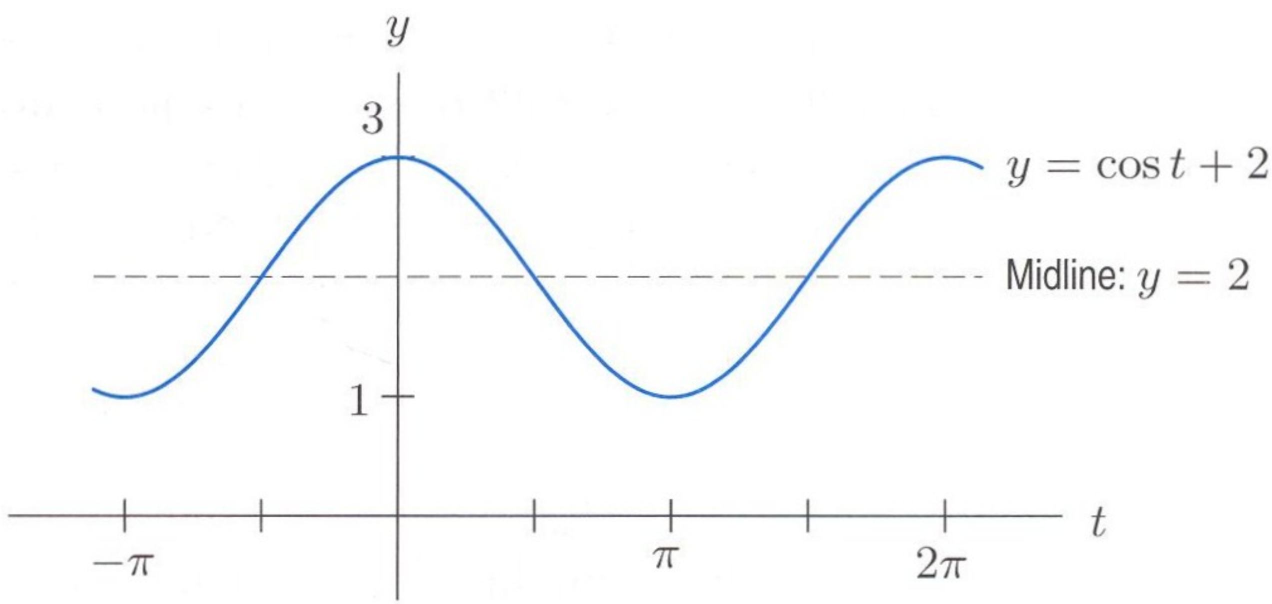


Figure 6.43: The graph of $y = \cos t + 2$ and its midline $y = 2$

Notice the midline is also shifted 2 units up.

London Eye Graph

Graph the ferris wheel function giving your height, $h = f(\theta)$, in feet, above the ground as a function of the angle θ.

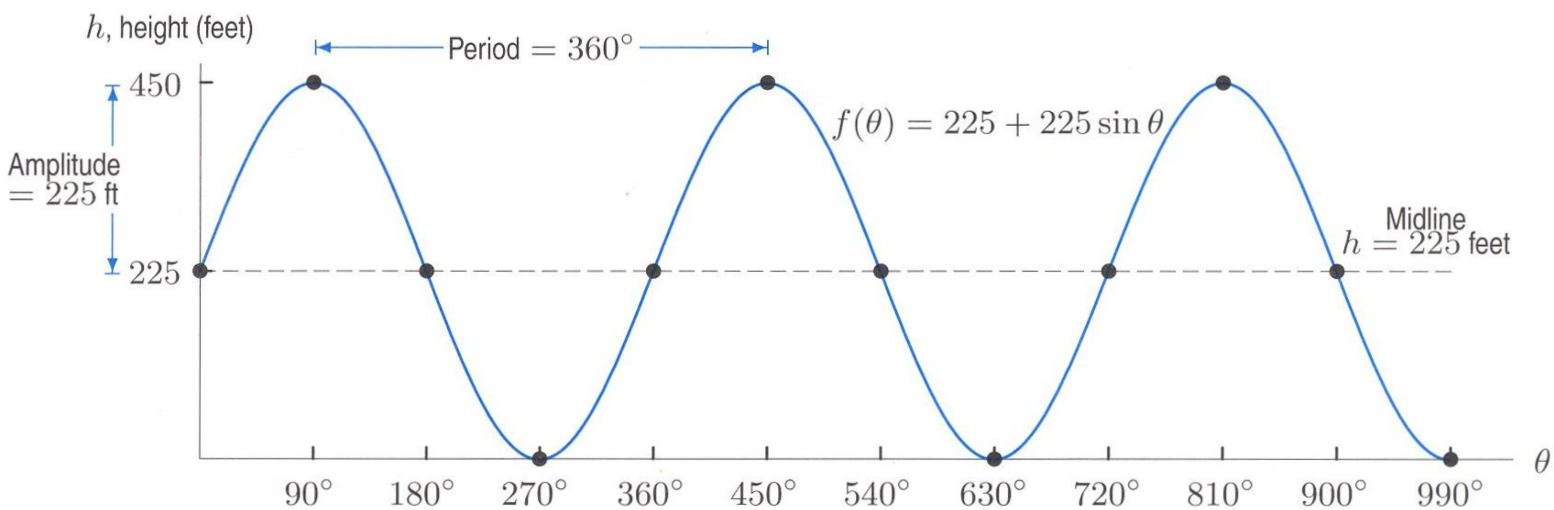
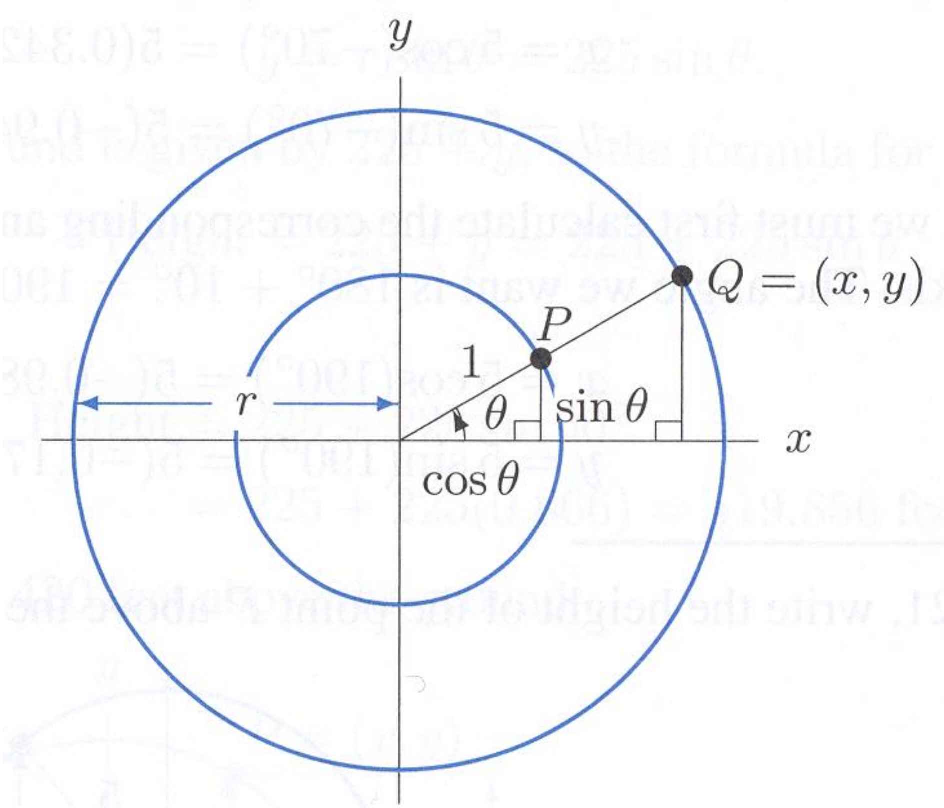


Figure 6.44: On the ferris wheel: Height, h, above ground as a function of the angle, θ

Coordinates of a Point on a Circle of Radius *r*

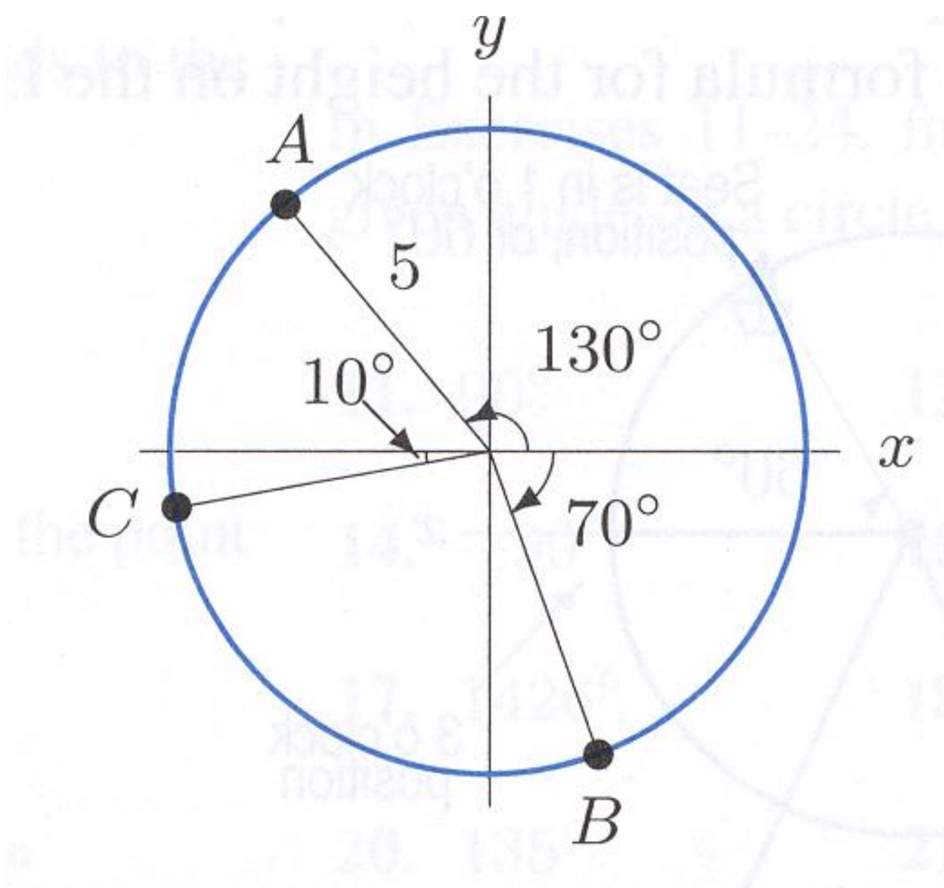


$$\frac{x}{\cos\theta} = \frac{r}{1} \quad \text{and} \quad \frac{y}{\sin\theta} = \frac{r}{1}$$

The coordinates (*x*, *y*) of the point *Q* are given by $x = r\cos\theta$ and $y = r\sin\theta$.

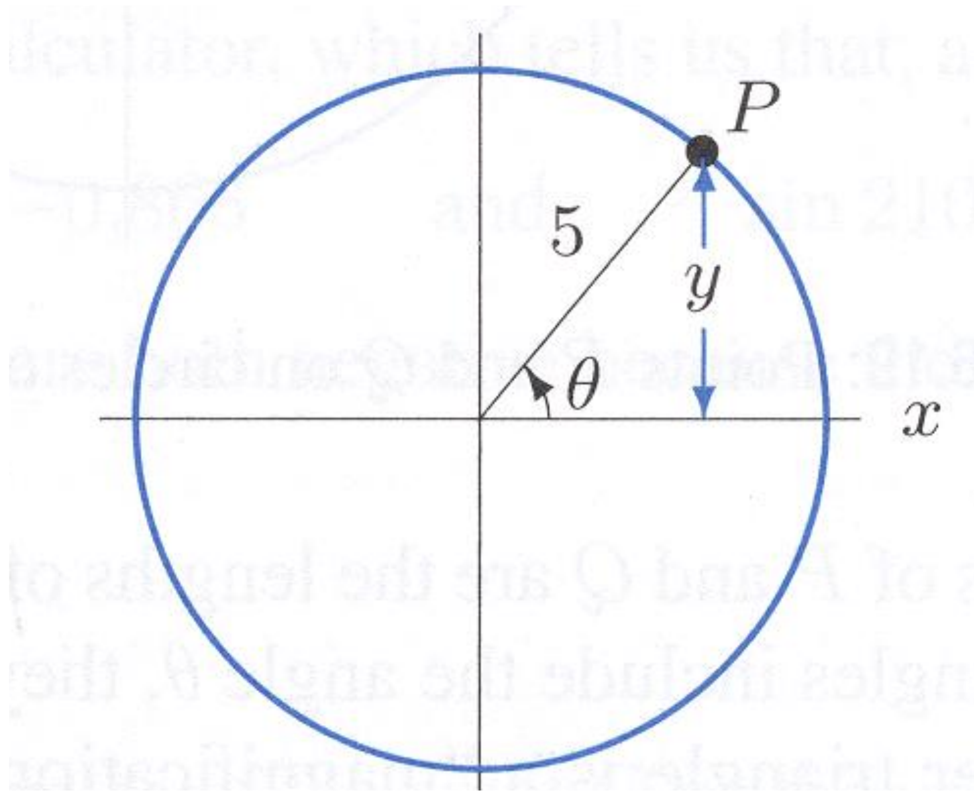
Calculations on a Circle of Radius 5

Find the coordinates of the points *A, B,* and *C* in the figure pictured to the right accurate to three decimal places.



Another Example Using Radius 5

In the figure to the right, write the height of the point *P* above the *x*-axis as a function of the angle θ. Also graph this function.



Height on the Ferris Wheel as a Function of Angle

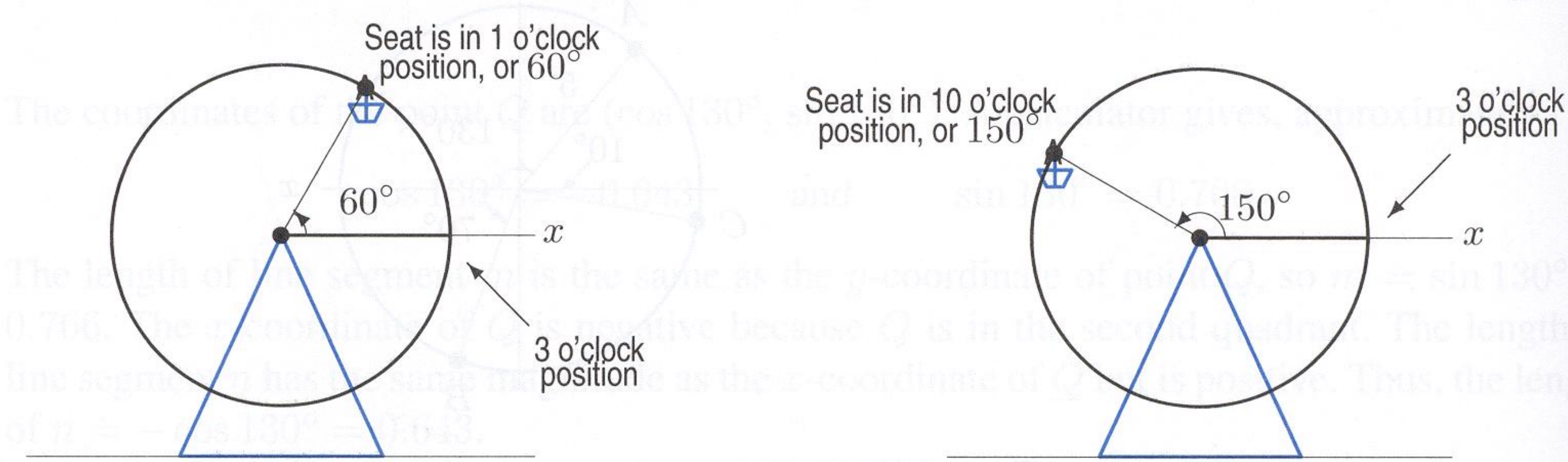
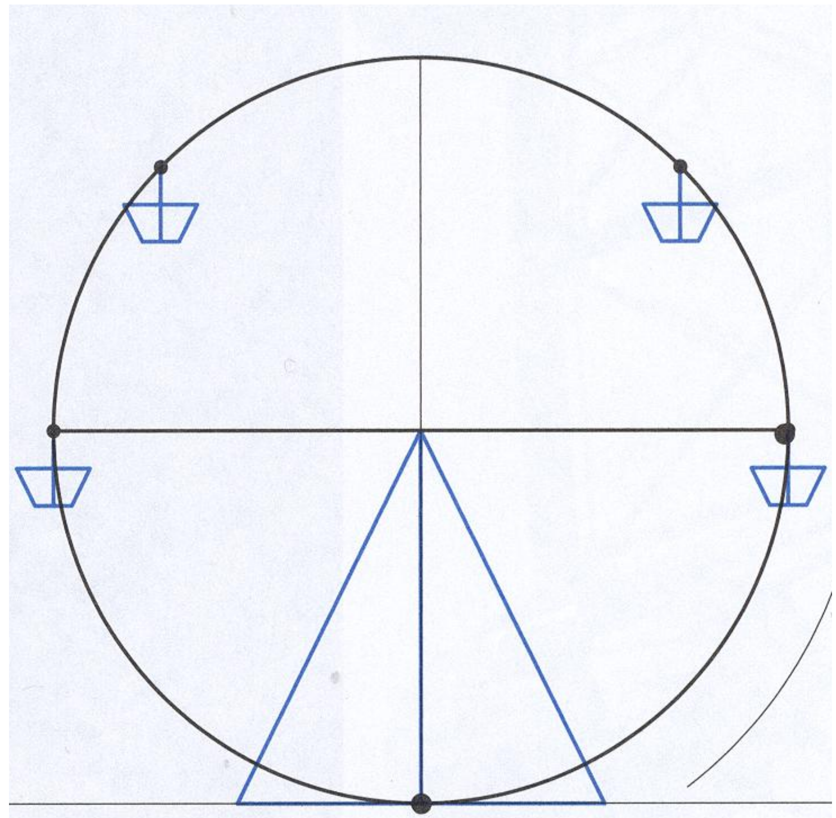


Figure 6.22: The 1 o'clock position forms a 60° angle with the positive x-axis, and the 10 o'clock position forms a 150° angle

Find your height above the ground for both of the angles θ pictured above.

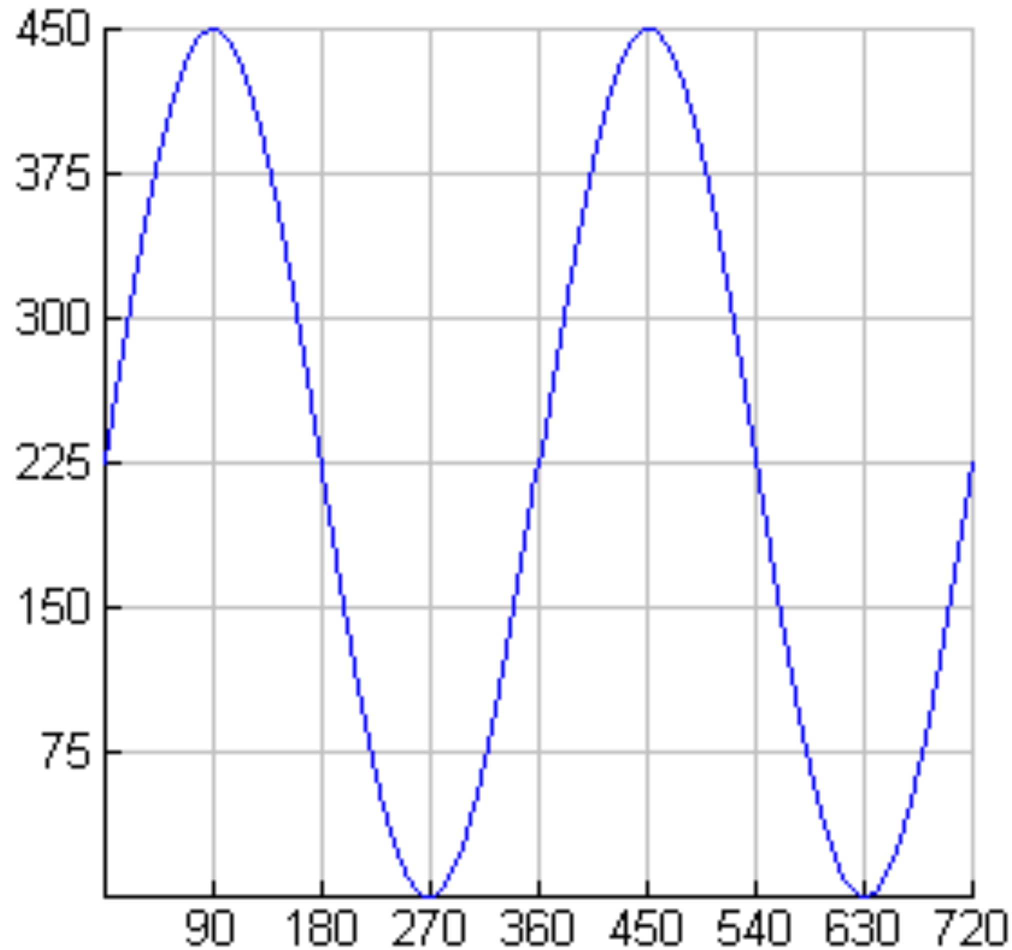
Heights on the London Eye

The London Eye has a radius of 225 feet. Find your height above the ground as a function of the angle θ measured from the 3 o'clock position.



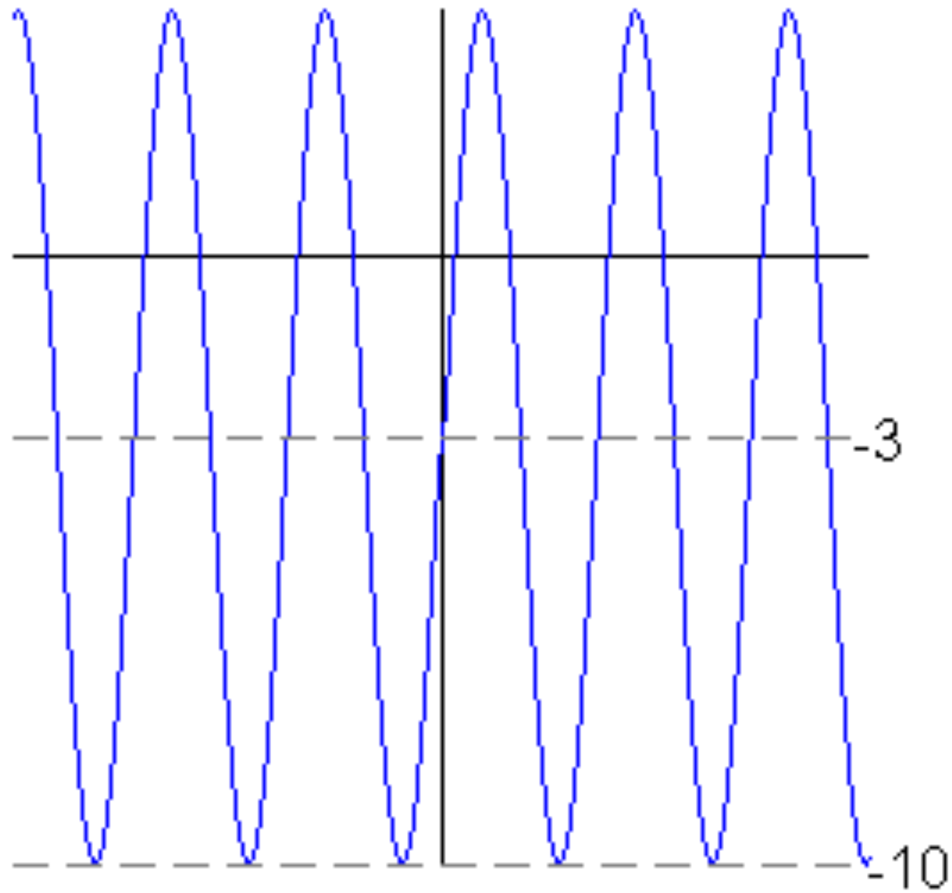
Graph for the London Eye

Graph the Ferris wheel function, found previously, giving your height, $h = f(\theta)$, in feet, above ground as a function of the angle θ. What are the period, midline, and amplitude?



Exercise #3

Find the midline and amplitude of the periodic function pictured to the right.



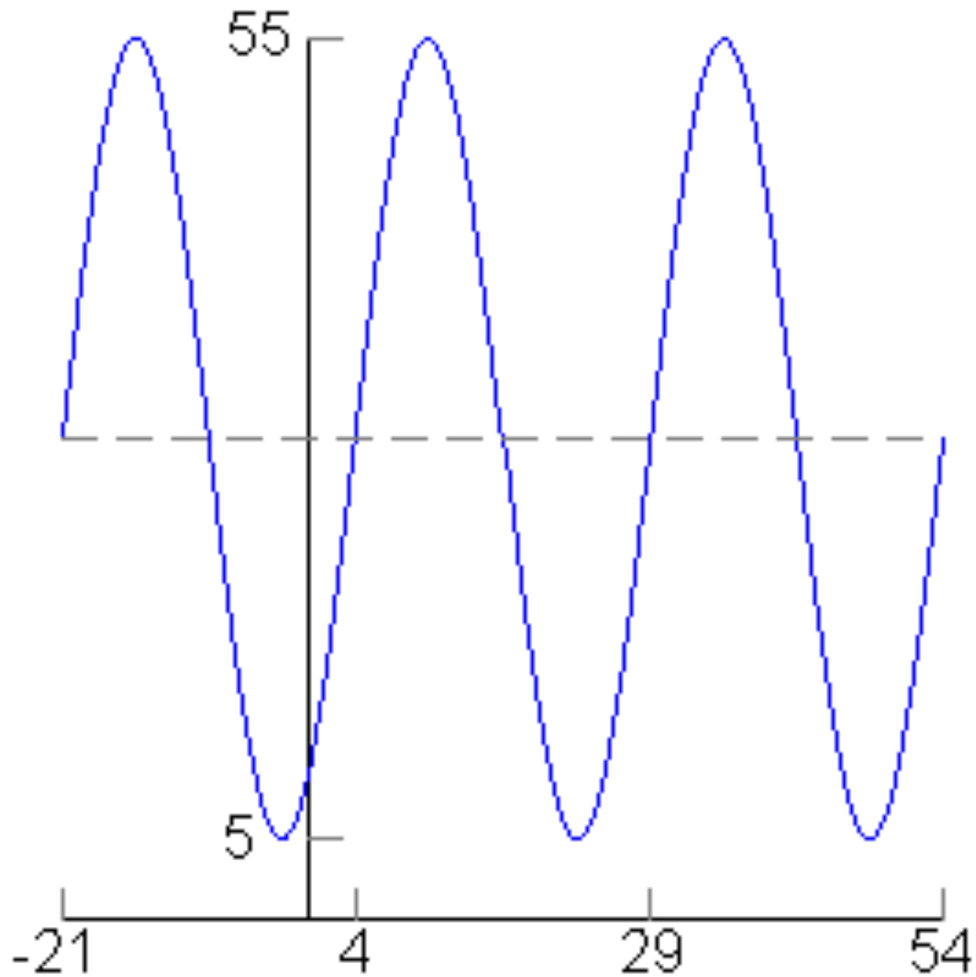
Exercises #11, #13, and #17

Find the coordinates of the point at the given angle on a circle of radius 3.8 centered at the origin.

11. -270°
13. 1426°
17. 225°

Problem #27

Estimate the period, midline, and amplitude of the periodic function pictured to the right.



Problem #34

A Ferris wheel is 20 meters in diameter and makes one revolution every 4 minutes. For how many minutes of any revolution will your seat be above 15 meters?