(1)

A health inspector must inspect each of five different restaurants {Raising Cane's®, Taco Bell®, PieWorks®, Subway® and Wendy's®}, twice, visiting one restaurant per day. The inspector is free to select the order in which to visit these restaurants, but cannot visit Taco Bell on two consecutive days, for obvious reasons. In how many different orders can the inspector visit these restaurants?

(2)

We write one n-digit number on a slip of paper. We include numbers beginning with 0's, for example when $n = 5$, one such number is 00158. Since the digits 0, 1, and 8 look the same upside down, and since 6 and 9 are interchanged when a slip of paper is turned upside down, 5-digit numbers such as 61891 and 16819 can share the same slip of paper. If we want to include all possible n-digit numbers but allow this kind of sharing, how many different slips of paper do we need?

(3)

Nine people are seated around a circular table for dinner, where they each order a unique dish. Unfortunately there is a mixup, and when the food is delivered, it turns everyone is served the food order of someone else. Prove that the table can be rotated so that at least two patrons are seated with their correct order.

(4)

Find a recurrence relation for the sequence of squares of Fibonacci numbers, and solve for the closed formula.

(5)

Prove the following general form of the Inclusion Exclusion Principle:

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i| - \sum_{i,j\,:\,1\le i<j\le n} |A_i \cap A_j| + \sum_{i,j,k\,:\,1\le i<j<k\le n} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n-1} |A_1 \cap \cdots \cap A_n|$$