## HW2 - Math 310 - Spring 2011 Mark Goadrich January 20, 2011

- (1) How many eight-bit strings have exactly two 1's? How many eight-bit strings have at least one 1?
- (2) We have a set of five distinct computer science books, three distinct mathematics books, and two distinct art books. In how many ways can these books be arranged on a shelf if the two art books are not together?
- (3) Prove the Inclusion-Exclusion Principle for three finite sets:  $|X \cup Y \cup Z| = |X| + |Y| + |Z| |X \cap Y| |X \cap Z| |Y \cap Z| + |X \cap Y \cap Z|$
- (4) How many integers between 1 and 10,000, inclusive, are multiples of 3 or 5 or 11 or 13 or any combination thereof?
- (5) Suppose that a pizza parlor features four specialty pizzas and pizzas with three or fewer unique toppings (no choosing bacon twice!) chosen from 17 available toppings. How many ways are there to select 4 different pizzas?
- (6) Find the number of rearrangements of the string 12345 in which none of the sequences 12, 23, 34, and 45 occur.
- (7) An exam has 12 problems. How many ways can (integer) points be assigned to the problems if the total of the points is 100 and each problem has to be worth at least five points?
- (8) We have piles of identical red, blue and green balls, where each pile contains at least 10 balls. How many ways can 10 balls be selected if exactly one red ball and at least one blue ball must be selected? In how many ways can 10 balls be selected if at most one red ball is selected?
- (9) Prove that the number of solutions to the equation  $x_1 + x_2 + x_3 = n, n \ge 3$  where  $x_1, x_2$  and  $x_3$  are positive integers, is  $\frac{(n-1)(n-2)}{2}$