HW3 - Math 310 - Spring 2011

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(1) **Prove that at a party where there are at least two people, there are two people who know the same number of other people there.**

(2) **An employee's time clock shows that she worked 81 hours over a period of 10 days. Show that on some pair of consecutive days, the employee worked at least 17 hours.**

(3) **Let** $D_i = \sum_{n=i,r=0}^{n=\lceil \frac{i+1}{2} \rceil, r=\lfloor \frac{i-1}{2} \rfloor} \binom{n}{r}$**, the sum of the shallow diagonals of Pascal's Triangle. Show that the sequence** D_i **is equal to the Fibonacci sequence.)**

(4) **Twelve basketball players, whose uniforms are numbered 1 through 12, stand around the center ring on the court in an arbitrary arrangement. Show that there are three consecutive players where the sum of their numbers is at least 20.**

(5) **Show that** $0 = \sum_{k=0}^{n} (-1)^k C(n, k)$

(6) **Professor Euclid is paid every other week on Friday. Show that in some month she is paid three times.**

(7) **Prove every sequence of** $n^2 + 1$ **distinct real numbers contains a subsequence of length** $n + 1$ **that is either strictly increasing or strictly decreasing.**

(8) **Show that in a group of 10 people on Facebook, there are either three mutual friends or four mutual strangers, and there are either three mutual strangers or four mutual friends.**

(9) **A bit string has even parity if it has an even number of 1's. How many bit strings of length** n **have even parity?**