HW4 - Math 310 - Spring 2011

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(1) **How many well-formed parenthesized expressions are there containing n distinct binary operators, $n+1$ distinct variables, and $n-1$ pairs of parentheses?**

(2) **Prove that for the Ackerman function, $A(m, n) > n$ for all $m \geq 0, n \geq 0$ by induction on m. The inductive step will use induction on n.**

(3) **Suppose that we have n dollars and that each day we buy either tape (\$1), paper (\$1), pens (\$2), pencils (\$2), or binders (\$3). Derive a recurrence relation where R_n is the number of ways of spending all the money and solve for a closed formula.**

(4) **Let $S_1 = 0, S_2 = 1, S_n = \frac{S_{n-1}+S_{n-2}}{2}$. Find a closed formula for this recurrence relation.**

(5) **The Lucas sequence is defined as $L_n = L_{n-1} + L_{n-2}, L_1 = 1, L_2 = 3$. Find a closed formula for this sequence, and show that $L_{n+2} = f_{n+1} + f_{n+3}$ where f_n is the Fibonacci sequence.**

(6) **In how many ways can a $2 \times n$ rectangular checkerboard be tiled using 1×2 and 2×2 pieces?**

(7) **A string that contains only 0s, 1s, and 2s is called a ternary string. Find a recurrence relation and initial conditions for the number of ternary strings that do not contain consecutive symbols that are the same, and solve for a closed formula.**

(8) **Add one more rule to the Towers of Hanoi puzzle, moving n disks from peg 1 to peg 3, such that you are not allowed to move a disk directly from peg 1 to peg 3. This means each move must be a move involving peg 2. Find a recurrence relation for the number of moves required to solve the puzzle for n disks, and solve this recurrence.**

(9) **Solve the following recurrence $a_n = -2a_{n-2} - a_{n-4}, a_0 = 0, a_1 = 1, a_2 = 2, a_3 = 3$.**