HW5 - Math 310 - Spring 2011

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March 17, 2011

(1) **Show that a graph with at least two vertices is bipartite if and only if it contains no odd-length cycles.**

(2) **An edge is called a bridge if when removed, the number of components increases. Prove that an edge e is a bridge of G if and only if e lies on no cycle of G.**

(3) **A directed acyclic graph (dag) is a simple directed graph with no cycles. Show that the maximum number of edges in an n-vertex dag is $\frac{n(n-1)}{2}$**

(4) **A vertex v in a connected graph G is an articulation point if the removal of v and all edges incident to v disconnects G. Show that a vertex v in a connected graph G is an articulation point if and only if there are vertices w and x in G having the property that every path from w to x passes through v.**

(5) **When does the complete graph K_n contain an Euler cycle? When does the complete bipartite graph $K_{m,n}$ contain an Euler cycle?**

(6) **Describe three sufficient properties of a connected graph such that the removal of any edge results in a graph that is not connected.**

(7) **Can a knight move around a chessboard and return to its original position making every move exactly once? (A move is considered to be made when the move is made in either direction.)**

(8) **Let v and w be distinct vertices in K_n. Let p_m denote the number of paths of length m from v to w in $K_n, 1 \leq m \leq n$. Derive a recurrence relation and find an explicit formula for p_m.**

(9) **Let G be a graph where $|V| = n$ and $|E| < n - 1$. Prove that G is not connected.**