

**HW6 - Math 310 - Spring 2011**

Mark Goadrich

March 31, 2011

- (1) Show that if the incidence matrix of a graph  $G$  is a square matrix, then  $G$  has a cycle.
- (2) Let  $G_1, G_2, G_3$  be any three graphs that have four vertices and two edges. Show that at least two of these graphs are isomorphic.
- (3) Let  $G$  be a bipartite graph with disjoint vertex sets  $V_1$  and  $V_2$ . Show that if  $G$  has a Hamiltonian cycle,  $V_1$  and  $V_2$  have the same number of elements.
- (4) Show that if  $n \geq 3$ ,  $K_n$  contains a hamiltonian cycle.
- (5) Write an algorithm that find the lengths of the shortest paths between all vertex pairs in a simple, connected, weighted graph having  $n$  vertices in time  $O(n^3)$ .
- (6) Let  $A$  be the adjacency matrix of the graph  $K_{m,n}$ . Find a closed formula for the entries in  $A^j$ .
- (7) Define  $\bar{G}$  as the complement of a graph  $G$ , where the vertices are the same, but an edge exists in  $\bar{G}$  only when it does not exist in  $G$ . Show that if  $G$  is a simple graph, either  $G$  or  $\bar{G}$  is connected.
- (8) Show the steps of applying the Shortest Path algorithm for Exercises 2 and 5 in the text on page 412.
- (9) Describe in general the result of multiplying an incidence matrix  $B$  times  $B^T$  (the transpose of  $B$ ).